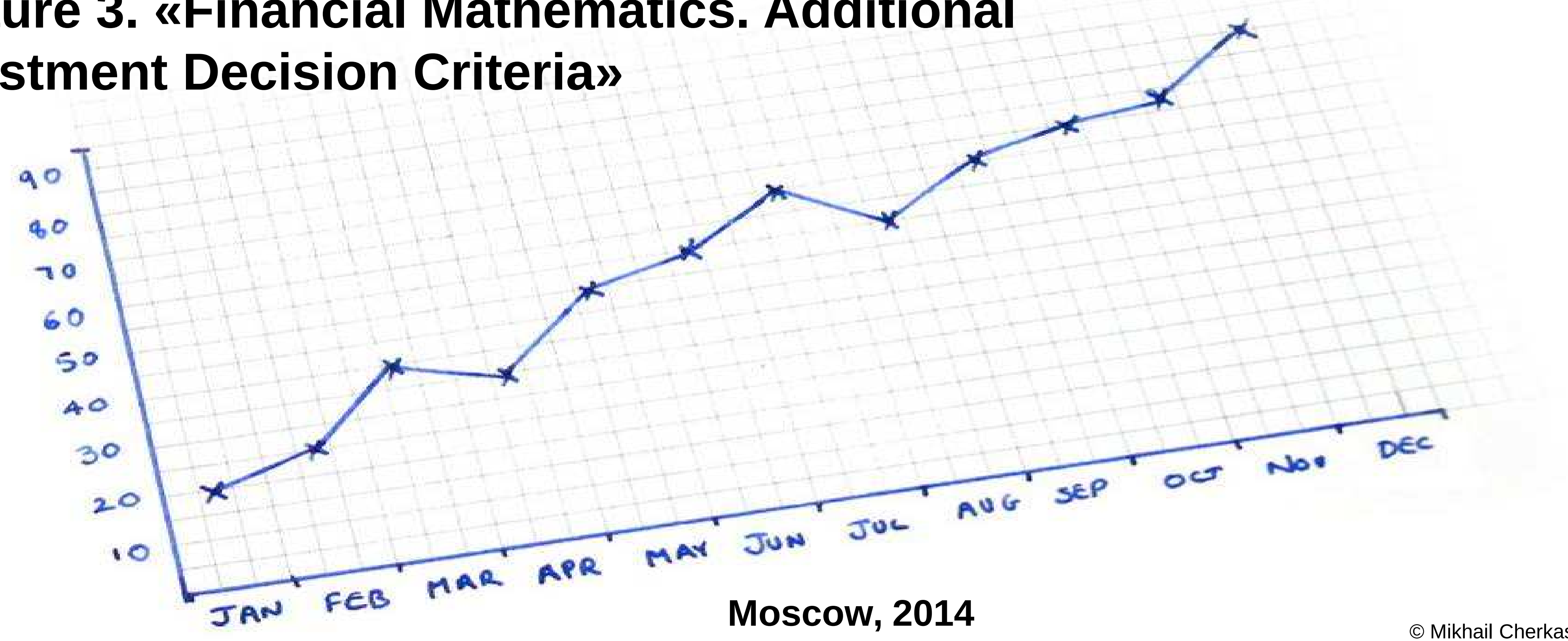
National Research University – Higher School of Economics

Investment Project Management

Lecture 3. «Financial Mathematics. Additional Investment Decision Criteria»



Moscow, 2014

Additional Investment Decision Criteria

Usually when we have mutually exclusive projects the key criterion for investment decision: the **NET PRESENT VALUE**, **IRR**, **PP**, **DPP**, **ANPV** and **DPI** are not sufficient. The deep analysis discovers that some projects or deviations inside the sole project can be distinguished only with using some additional models of valuation. Following instruments facilitate the search of the best project:

- **Duration** (Macauley Duration)
- **MNPV** (Modified Net Present Value)
- **MIRR** (Modified Internal Rate of Return)
- **NFV** (Net Future Value)
- **NRR** (Net Rate of Return)
- **MNRR** (Modified Net Rate of Return)
- **NTV** (Net Terminal Value)
- **WACC** (Weighted Average Cost of Capital).

Duration

The additional technique used for Project's timing measuring (and consequently, the investment decision creation) is **Duration** or Macauley duration. The difference between Duration and other similar methods is as follows:

Payback as a technique fails to take into account the time value of money and any cash flows beyond the project date. But it is used by many firms as a coarse filter of projects at initiating stage.

Discounted Payback does surmount the first difficulty but not the second in that it is still possible for projects with highly negative terminal cash flows to appear attractive because of their initial favourable cash flows. Conversely, discounted payback may lead a project to be discarded that has highly favourable cash flows after the payback date.

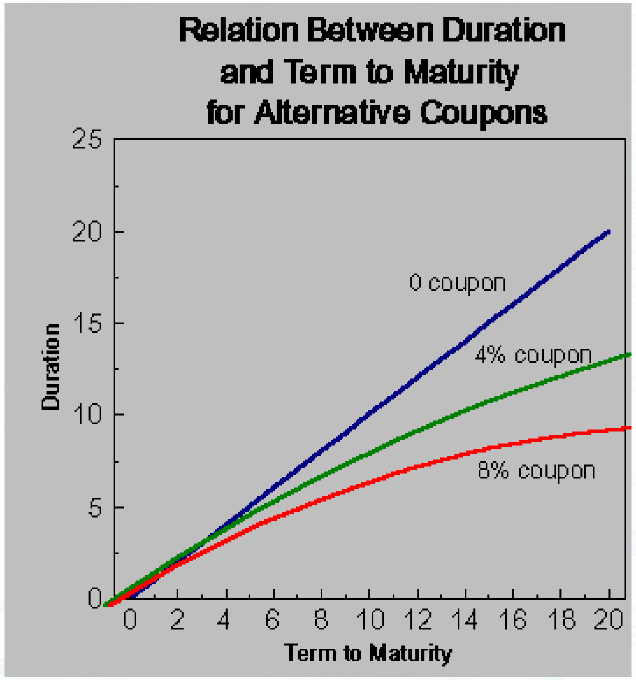
Duration measures either the average time to recover the initial investment (if discounted at the project's internal rate of return) of a project, or to recover the present value of the project if discounted at the cost of capital. Duration captures both the time value of money and the whole of the cash flows of a project. It is also a measure which can be used across projects to indicate when the bulk of the project value will be captured.

Duration

The concept of **Duration** is difficult for understanding. It demonstrates additional characteristic of Project's payback and the dependence of payback to the discount rate value.

The formula for calculation of the Duration (**D**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t) and the discount rate (**r**) is:

$$Duration\ (D) = \frac{\sum_{t=1}^{n}\left[t \times \frac{CF_t}{(1+r)^t}\right]}{\sum_{t=1}^{n}\frac{CF_t}{(1+r)^t}}$$



The easiest way for Macauley duration understanding is an analysis of bonds return within the scope of coupon rate changing.

Duration

The **Duration** is calculated as follows:

D (Duration)	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
NCF (Net Cash Flow) after tax	*12%*	-500,00	100,00	200,00	300,00	400,00	500,00	1 500,00
Period Number			1	2	3	4	5	
DCF **since period Y1**	*12%*		89,29	159,44	213,53	254,21	283,71	1 000,18
DCF multiplied to Period Number (**Y1** is period No 1)		-500,00	89,29	318,88	640,60	1 016,83	1 418,57	3 484,16
D (Duration)								3,48

The **Duration** can't be properly calculated when the cash flows since the period Y1 are alternate.

The Duration is considered exactly in case when the Project is financed by borrowed funds. In this case the Durations of cash inflows have to be equal or less than the Durations of loans/bonds repayments. Macaulay Duration is a measure of the elasticity of the price of the cash flow versus the interest rates and maturities of loans.

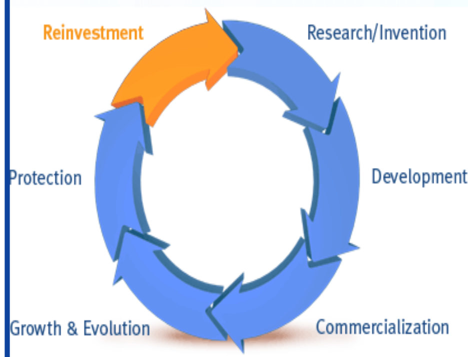
Frederic Macauley
1882 – 1970.

Modified Net Present Value

The modern technique used for Project's profitability measuring is **Modified NPV** (**MNPV**) or Modified Net Present Worth (MNPW). MNPV gives an opportunity to evaluate a project when the firm's opportunity rate of reinvestment is different from its financing rate.

The formula for calculation of the Modified Net Present Value (**MNPV**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t), the discount rate (**r**) the re-investment rate (**d**) is:

$$MNPV = \frac{\sum_{t=1}^{n} CF_t \times (1+d)^{n-t}}{(1+r)^n} - \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}$$



Modified Net Present Value

Modified NPV (MNPV) calculation:

MNPV	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
NCF (Net Cash Flow) after tax		**-5 000**	**3 845**	**-300**	**6 920**	**8 691**	**10 477**	**24 633,16**
Period Number		0	1	2	3	4	5	
Discount factor	*12%*	1,00	0,89	0,80	0,71	0,64	0,57	
Reinvestment rate (Compound factor)	*10%*	1,61	1,46	1,33	1,21	1,10	1,00	
Reinvested Positive Cash		**0,00**	**5 629,46**	**0,00**	**8 373,40**	**9 560,03**	**10 477,06**	**34 039,95**
Discounted Investment (all negative cash flows)		**5 000**	**0**	**239**	**0**	**0**	**0**	**5 239,16**
Discounted Positive Reinvested Cash		**Discount factor is (1+Hurdle Rate)^(Total number of periods)**						**19 315,18**
MNPV (Modified Net Present Value)								**14 076,03**

Discounting of the sum of entire re-invested cash is done at fixed rate equal to:

= (1 + Hurdle rate) raised to the power n (= total Number of periods).

Discounting of negative cash (investments) is done (according to the classic model at rate = WACC). In our case we use the Hurdle rate for Outlay discounting.

Modified Internal Rate of Return

Modified NPV (**MNPV**) calculation gives an idea of **Modified Internal Rate of Return** (**MIRR**) or RIRR (Reinvestment-rate adjusted Internal Rate of Return). But in order to avoid the problems generated by conventional IRR (multiple IRR, no IRR cash flows) the formula is ideologically changed:

The formula for calculation of the Modified Internal Rate of Return (**MIRR**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t), the discount rate (**r**) the re-investment rate (**d**) is:

$$MIRR: \sum_{t=0}^{n} \frac{I_t}{(1+r)^t} = \frac{\sum_{t=1}^{n} CF_t \times (1+d)^{n-t}}{(1+MIRR)^n}$$

or

$$MIRR = \sqrt[n]{\frac{FV\ (positive\ cash\ flow, reinvestment\ rate)}{-PV\ (negative\ cash\ flow,\ hurdle\ rate)}} - 1$$

Modified Internal Rate of Return

Modified IRR (MIRR) calculation:

MIRR	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
NCF (Net Cash Flow) after tax		**-5 000**	**3 845**	**-300**	**6 920**	**8 691**	**10 477**	**24 633,16**
Period Number		0	1	2	3	4	5	
Discount factor	*12%*	1,00	0,89	0,80	0,71	0,64	0,57	
Reinvestment rate (Compound factor)	*10%*	1,61	1,46	1,33	1,21	1,10	1,00	
Reinvested Positive Cash		**0**	**5 629**	**0**	**8 373**	**9 560**	**10 477**	**34 039,95**
Discounted Investment (all negative cash flows)		**5 000**	**0**	**239**	**0**	**0**	**0**	**5 239,16**
MIRR (Modified Internal Rate of Return)		=(Reinvested positive cash/Discounted Outlay)^(1/Number of periods)-1						**45,394%**

45,394%	Calculated using "**МВСД**" function, English version "**MIRR**"

MIRR Hurdle Rate (Modified Internal Rate of Return with reinvestment at Hurdle Rate)	*12%*	**46,156%**

Discounting of negative cash (investments) is done (according to the classic model at rate = WACC). In our case we use the Hurdle rate for Outlay discounting.

Special case: XNPV & XIRR

Excel gives special instrument for calculation of NPV and IRR for not even periods when cash flow occurs. These functions are named **XNPV** and **XIRR**.

XNPV, XIRR	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
NCF (Net Cash Flow) after tax		-5 000	-3 845	-300	6 920	-8 691	10 477	-438,71
Dates of transactions		19.08.2016	21.09.2016	25.09.2016	30.09.2016	07.10.2016	19.10.2016	

XNPV (Excel function for calculation of NPV for non-periodic cash flows)	**-551,09**	Calculated using "**ЧИСТНЗ**" function, English version "**XNPV**"
XIRR (Excel function for calculation of IRR for non-periodic cash flows)	**-34,307%**	Calculated using "**ЧИСТВНДОХ**" function, English version "**XIRR**"

The formula for calculation of the **XNPV** and XIRR with given future cash flows for each period d_t (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t) and the discount rate (**r**) is:

$$XNPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^{\frac{d_t - d_1}{365}}}; \quad XIRR:\ 0 = \sum_{t=1}^{n} \frac{CF_t}{(1+XIRR)^{\frac{d_t - d_1}{365}}}$$

Net Future Value

For each cash sum in the cash flow the **NPV** discounts it back to what would have to be invested to produce the sum and the **Net Future Value** (**NFV**) compounds it forward to what it would produce if invested immediately it is received. In other words, if you invest the NPV for *n* years at r % the final sum is the NFV.

The formula for calculation of the Net Future Value (**NFV**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t) and the discount rate (**r**) is:

$$NFV = \sum_{t=0}^{n} CF_t \times (1+r)^t$$

or

$$NFV = NPV \times (1+r)^n$$

Net Future Value

Net Future Value (NFV) calculation:

NFV	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
NCF (Net Cash Flow) after tax		-500	**100**	**200**	**300**	**400**	**500**	**1 000,00**
NPV (only positive cash flows)			**89,29**	**159,44**	**213,53**	**254,21**	**283,71**	**1 000,18**
Period Number			1	2	3	4	5	5
Hurdle rate and Compound factor	***12%***	1,0000	1,1200	1,2544	1,4049	1,5735	1,7623	
NFV (Net Future Value)			157	281	376	448	500	
			Invested for 4 years	Invested for 3 years	Invested for 2 years	Invested for 1 year	Invested for 0 years	**1 762,66**

NFV (Net Future Value) as function of NPV		**1 762,66**	**=NPV*(1+Hurdle rate)^Number of periods**

With **negative** cash flows (investments) since **Y1** **NFV** can't be calculated properly.

Net Rate of Return

The **Net Rate of Return** (**NRR**) – average rate of return used in combination with Discounted Profitability Index (DPI) or Benefit to Cost Ratio (BCR). It reflects the profitability of the net discounted investment.

The formula for calculation of the Net Rate of Return (**NRR**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t) and the discount rate (**r**) is:

$$NRR = \frac{NPV}{\left| \sum_{t=0}^{n} \frac{I_t}{(1+r)^t} \right| (modulus)} \times 100\%$$

or

$$NRR = (1 - DPI) \times 100\%$$

Net Rate of Return

The **Net Rate of Return** (**NRR**) calculation:

NRR	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
Cumulative DCF = **NPV**	*12%*	-5 000,00	-1 566,96	-1 806,12	3 119,52	8 642,76	14 587,73	14 587,73
Initial Investment Discounted		-5 000,00	0,00	-239,16	0,00	0,00	0,00	-5 239,16
NRR (Net Rate of Return)								278,44%
DPI (Discounted Profitability Index)								3,7844

$$NRR = (1 - DPI) \times 100\%$$

The NRR is a derivative (not in mathematical sense) of **DPI** but sometimes this ratio can be useful.

Modified Net Rate of Return

The **Modified Net Rate of Return** (**MNRR**) is the analogue of Net Rate of Return (NRR) which uses Modified Net Present Value (MNPV) as re-invested NPV. It reflects the profitability of discounted outlay with regular re-investment of the positive cash flows at the rate different to demanded rate of discounting.

The formula for calculation of the Modified Net Rate of Return (**MNRR**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t), the investments for each period (I_t) and the discount rate (**r**) is:

$$MNRR = \frac{MNPV}{\sum_{t=0}^{n} \frac{I_t}{(1+r)^t}} \times 100\%$$

Modified Net Rate of Return

The **Modified Net Rate of Return** (**MNRR**) calculation:

MNRR	Rate	Y0	Y1	Y2	Y3	Y4	Y5	Total for the period
MNPV (Modified Net Present Value)								14 076,03
Discounted Investment (all negative cash flows)		5 000	0	239	0	0	0	5 239,16

MNRR (Modified Net Rate of Return)	268,67%

Weighted Average Cost of Capital

The **Weighted Average Cost of Capital** (**WACC**) is the minimum level of return that a company has to earn in order to satisfy its creditors, owners, and other providers of capital, or they will invest elsewhere. The cost of Equity is calculated from annual dividends repayments. The cost of Debt is integrated from the interest rates of all types of borrowings, including: preferred stocks, leasing, bonds and other sources.

The formula for calculation of the Weighted Average Cost of Capital (**WACC**) with given amounts of Equity and Debt future with respective rates of dividends repayments and interest rates on all sorts of debt is*****:

$$WACC = \left(\frac{Equity}{Cap}\right) \times Dividend\,Yield + \left(\frac{Debt}{Cap}\right) \times Interest\,Rate \times (1 - Tax\,Rate)$$

*****The formula is interpreted for the Project under the control of private company. Dividend Yield represents all cash repayments to the stakeholder.

Weighted Average Cost of Capital

The **Weighted Average Cost of Capital** (**WACC**) calculation:

WACC	Rate	Y0	Y1	Y2	Y3	Y4	Y5
Equity		**5 000**	**7 000**	**12 000**	**12 000**	**12 000**	**12 000**
Straight debt (average for the year)		**0**	**4 932**	**5 918**	**6 904**	**7 890**	**8 877**
Total Capital		**5 000**	**11 932**	**17 918**	**18 904**	**19 890**	**20 877**
Share of net profit paid as dividends	***25%***	25,00%	25,00%	25,00%	25,00%	25,00%	25,00%
Interest on debt	***6,10%***	0,00%	8,00%	7,00%	6,00%	5,00%	4,50%
Corporate tax rate	***20%***						
WACC (Weighted Average Cost of Capital)		**25,00%**	**17,31%**	**18,59%**	**17,62%**	**16,67%**	**15,90%**
							18,5163%

WACC is recommended by all valuation models as a Hurdle Rate for calculation of all key profitability ratios.

Net Terminal Value

Net Terminal Value (**NTV**) is the estimated Company (Project) value beyond the horizon of projection, i.e. the valuation of all future cash flows with expected **stable** growth rate forever. For sure, such a valuation doesn't consider numerous risks of the global/country/regional economy, interest rates changes, etc. and, consequently, can't be recognized as a true Company/Project value. But this ratio is used very often for assessments.

The formula for calculation of the Net Terminal Value (**NTV**) with given future cash flow for the last period of projection – the year **t** (CF_t), the rate of future growth (**g**) and the discount rate (**r**) is:

$$NTV = \frac{CF_t \times (1 + g)}{(r - g)}$$

Net Terminal Value

Net Terminal Value (**NTV**) is used in Gordon model for calculation of Company value with stable growth rates linked to the total economy's growth.

NTV	Rate	
CF (Cash Flow) within last period of projection		**5 982,01**
Hurdle rate	***12%***	
Growth rate	***5%***	
Net Terminal Value (NTV)		**89 730,09**

The Net Terminal Value (**NTV**) is a result of transformation of the Perpetuity Growth Method applied to cash flow estimations for the Project with given future cash flow for the last period of projection – the year **t** ($FCFF_t$) and the existing cost of capital (**WACC**):

$$Terminal\,Value = \sum_{t=1}^{t=\infty} \frac{FCFF_t}{(1 + WACC)^t}$$

Multiple Discounting and Compounding Rates

DPI with variating hurdle rates	$DPI = \dfrac{\sum_{t=0}^{n} \dfrac{CF_t}{\prod_{i=1}^{t}(1+r_i)}}{\sum_{t=0}^{n} \dfrac{I_t}{\prod_{i=1}^{t}(1+r_i)}}$
DPP with variating hurdle rates	$DPP = n, when \sum_{t=1}^{n} \dfrac{I_t}{\prod_{i=1}^{t}(1+r_i)} > I_0$
Duration with variating hurdle rates	$Duration = \dfrac{\sum_{t=1}^{n} t \times PV_t}{\sum_{t=1}^{n} PV_t}, where\ PV_t = \dfrac{CF_t}{\prod_{i=1}^{t}(1+r_i)}$
MIRR Hurdle Rate (with reinvestment at variating Hurdle Rate)	$MIRR_{hurdle}: \sum_{t=0}^{n} \dfrac{I_t}{\prod_{i=0}^{t}(1+r_i)} = \dfrac{\sum_{t=1}^{n} CF_t \times \prod_{i=1}^{n-t}(1+r_{i+1})}{(1+MIRR_{hurdle})^n}$
MIRR with variating discount and reinvestment rates	$\sum_{t=0}^{n} \dfrac{I_t}{\prod_{i=0}^{t}(1+r_i)} = \dfrac{\sum_{t=1}^{n} CF_t \times \prod_{i=1}^{n-t}(1+d_{i+1})}{(1+MIRR)^n}$
MIRR Hurdle Rate (with reinvestment at constant Hurdle Rate)	$MIRR_{hurdle}: \sum_{t=0}^{n} \dfrac{I_t}{(1+r)^t} = \dfrac{\sum_{t=1}^{n} CF_t \times (1+r)^{n-t}}{(1+MIRR_{hurdle})^n}$
MNPV with variating hurdle rates	$MNPV = \dfrac{\sum_{t=1}^{n} CF_t \times \prod_{i=1}^{n-t}(1+d_{i+1})}{\prod_{t=1}^{n}(1+r_t)} - \sum_{t=o}^{n} \dfrac{I_t}{\prod_{i=0}^{t}(1+r_i)}$
NPV with variating hurdle rates	$NPV = \sum_{t=1}^{n} \dfrac{CF_t}{\prod_{i=1}^{t}(1+r_i)} - \sum_{t=0}^{n} \dfrac{I_t}{\prod_{i=0}^{t}(1+r_i)}$

Project/Company Valuation

Company Valuation based on WACC discount rate								
CF (Cash Flow)		**-5 000**	**3 285**	**-1 071**	**6 461**	**8 391**	**10 542**	**22 608,39**
WACC (Weighted Average Cost of Capital)		**25,00%**	**17,31%**	**18,59%**	**17,62%**	**16,67%**	**15,90%**	**18,52%**
NTV (Net Terminal Value)								**46 470,62**
Hurdle rate	***18,52%***							
Growth rate	***5%***							
Number of period		1	2	3	4	5	6	
Discount factor (special rate)		0,9186	0,7751	0,6540	0,5518	0,4656	0,3928	
Discounted Cash Flow (special rate)		**-4 592,84**	**2 546,06**	**-700,40**	**3 565,12**	**3 906,76**	**4 141,50**	**8 866,20**
Company Value								**27 121,93**

WACC is used as a multiple variable discount rate.

The Valuation of the Company (**V**) based on Net Terminal Value (**NTV**) with with given future cash flows for each period **t** (among **n** periods) starting since year 1 (CF_t) and the discount rate (**r**) which is taken as an existing cost of capital (**WACC**) based on Gordon's model:

$$V = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^{t-0,5}} + \frac{NTV}{(1+r)^{n-0,5}}$$