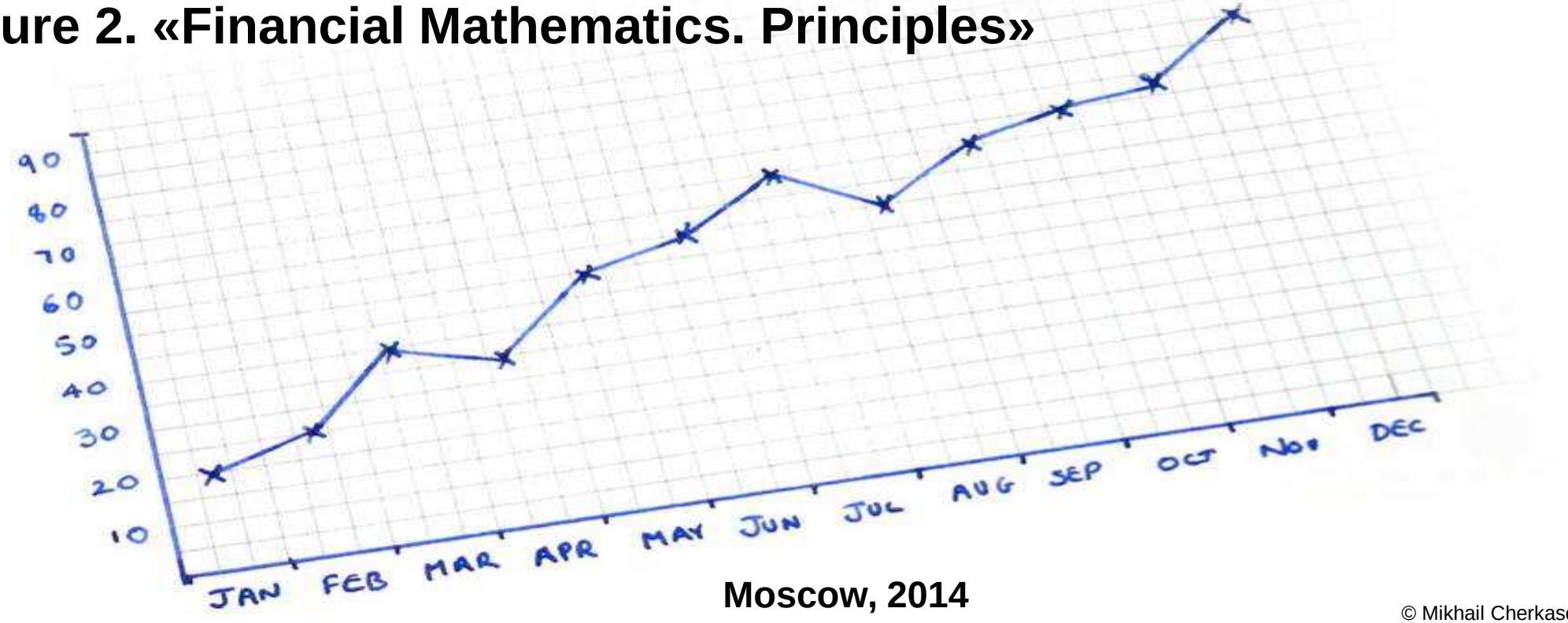
National Research University – Higher School of Economics

Investment Project Management

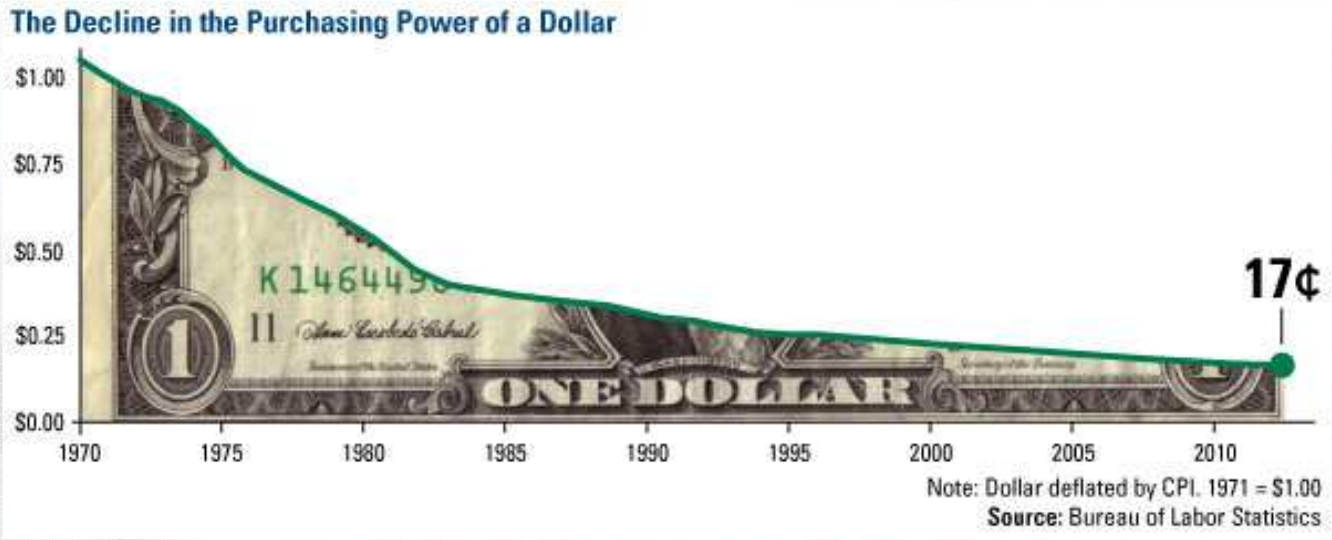
Lecture 2. «Financial Mathematics. Principles»



Moscow, 2014

What the financial math does stand on?

The only hypothesis lays at the basement of modern financial mathematics: “The dollar **TODAY** is worth more!”



This principle named “**The Time Value of Money**” is used for explanation of Present Value phenomenon and all other existing valuation models.

What the financial math does stand on?

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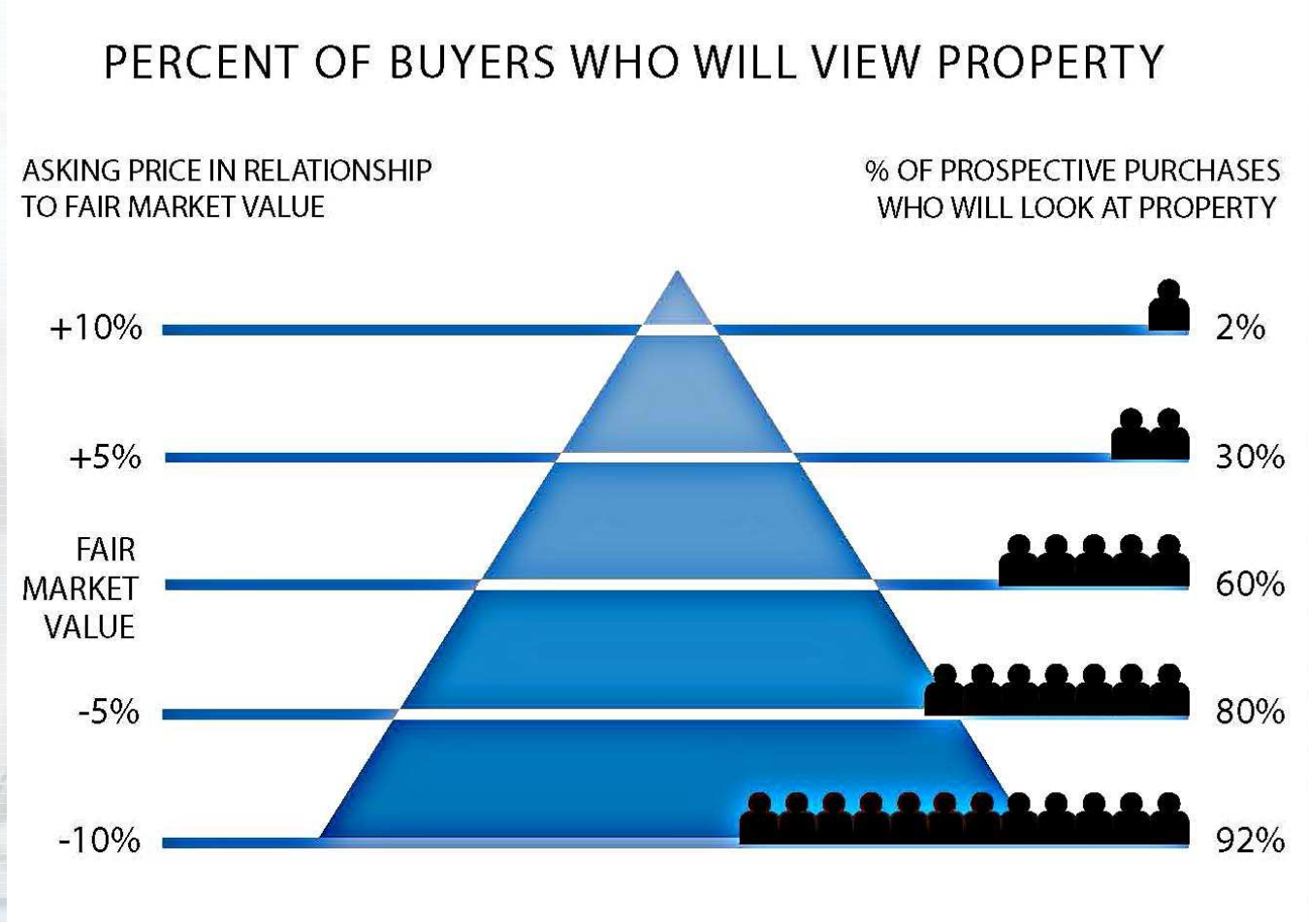
Another disparity which generates various math models – it's a conflict between the '**Price**' and the '**Value**'.

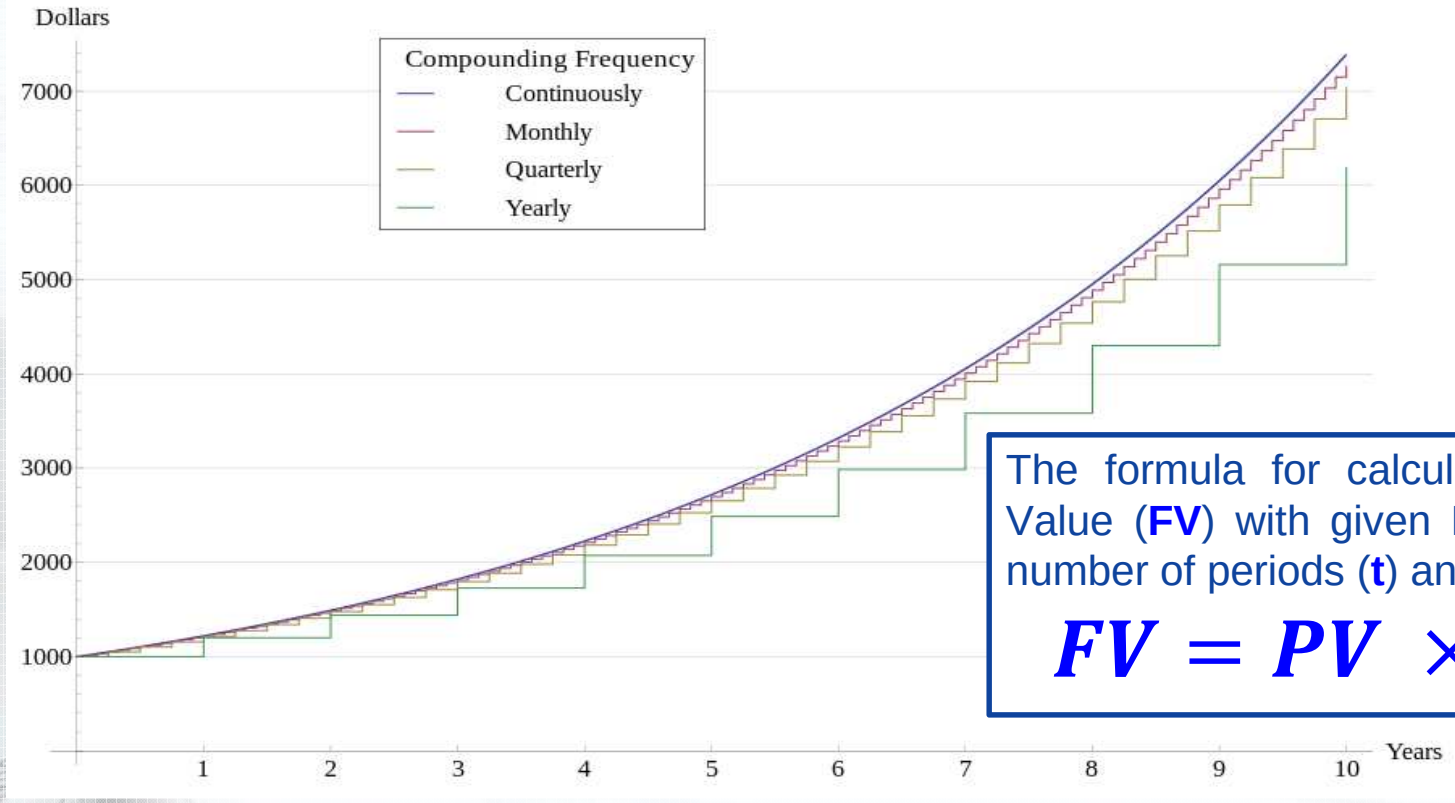


What the financial math does stand on?

The key idea is that today's dollar will cost tomorrow more as far as it can be invested today and can generate some additional cash till tomorrow. Consequently, there's today's and tomorrow's value of a dollar. We can say "**PRESENT VALUE**" and "**FUTURE VALUE**".

What the financial math does stand on?

The process of generating of **FUTURE VALUE** by investing money is named **COMPOUNDING**.



The formula for calculation of the Future Value (**FV**) with given Present Value (**PV**), number of periods (**t**) and interest rate (**r**) is:

$$FV = PV \times (1 + r)^t$$

The more frequent are the acts of investing the higher amount of the additional value is generated.

What the financial math does stand on?

The process of generating of **FUTURE VALUE** by investing money is named **COMPOUNDING**.

Timmy's problem

How to calculate quickly the number of re-investment periods needed for turning $10'000 to $110'000 when the interest rate is given?

Interest (Compounding) Rate	***10%***		
Present Value	***10 000***		
Future Value	***110 000***		
Number of periods needed	***?***		
Number of periods needed	**25,16**		

The formula for calculation of the number of periods of re-investment when the Present Value is turning to Future Value with given interest rate is:

$$n = \frac{ln(\frac{FV}{PV})}{ln(1+r)}$$

What the financial math does stand on?

The opposite task: to find out the Present Value when we know the Future Value. This process is named **DISCOUNTING**.

Sheila's problem

How to calculate quickly the amount to be invested to obtain $11'000 in 2 years when the interest rate is given?

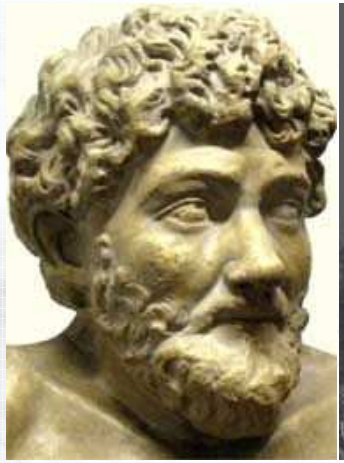
Interest (Compounding) Rate	***10%***
Present Value	***?***
Future Value	***11 000***
Number of periods	***2***
Amount to be invested	**9090,91**

The formula for calculation of the Present Value (**PV**) with given Future Value (**FV**), number of periods (**t**) and interest rate (**r**) is:

$$PV = \frac{FV}{(1+r)^t}$$

What the financial math does stand on?

Let’s return to a basic category of financial mathematics – the **PRESENT VALUE**. How much do tomorrow’s dollars cost today for those who prefer spend money immediately and those who prefer to keep them for a long?



Aesop

Jean de La Fontaine

Ivan A. Krylov

What the financial math does stand on?

The Ant and the Grasshopper			
The Ant is idustrous guy and he doesn't like to spend money.			
The Grasshopper is a prodigal (risky spender) and she loves to throw her money aroun			
The Ant deposits his money at the bank.			
The Grasshopper borrows any amount of money at the bank at any moment.			
The rates on loans and deposits are equal	***10%***		
The amount to be invested	***100***		
The investment rate of return	***15%***		
	Y0	***Y1***	***Total***
Ant's actions	**-100**	**115**	**15**
Extra-value received in comparison with bank deposit	**4,55**		**5,00**
Grasshoppers's actions			
Bank's loan (with interest included)	104,55	-115,00	
Investing	-100,00	115,00	
Spending immediately	**4,55**		

What are the Goals of economic analysis?

OR

?

Increase in the Company's **Value** generates the positive effect for the stakeholders. Increase in the **Profits** can mean: 1) growth of this year profit to the prejudice of further years profits; 2) growth of the profit without dividends repayment; 3) various technical methods of profit calculation. That's why the key goal of the business should be an **increase of Company's Value**.

Valuation ≠ Accounting reports analysis

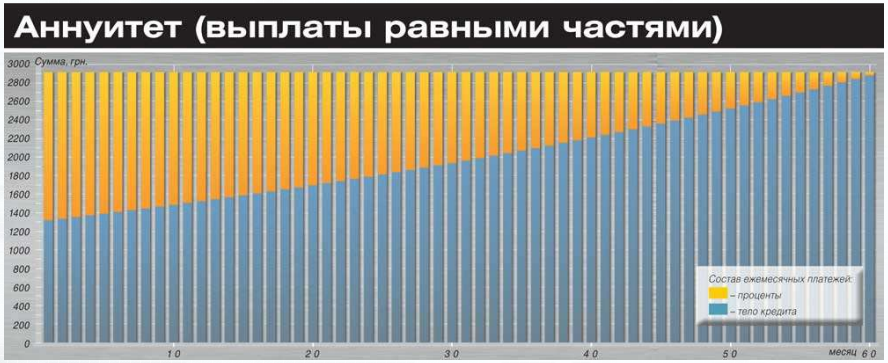
The valuation is based on cash flows. Accounting report analysis is based on revenues, costs and profit/income.

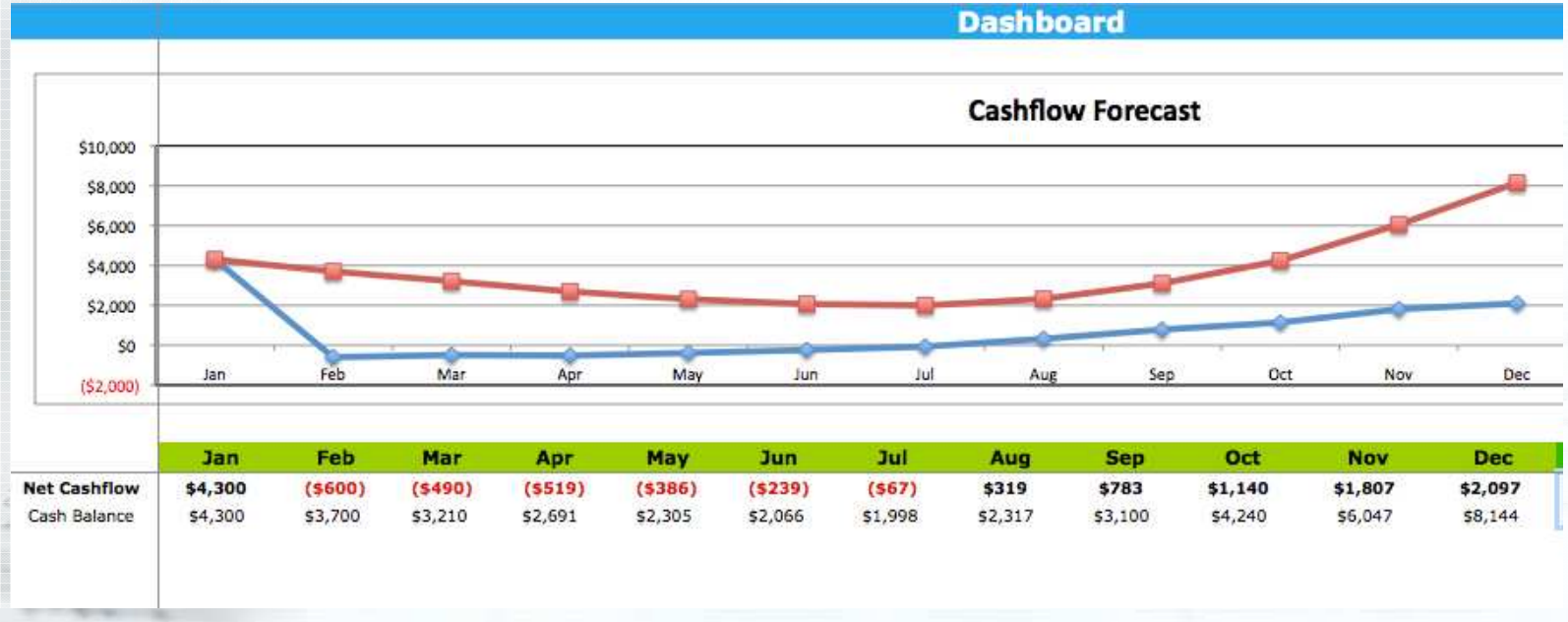
Various definitions

Valuation – is finished process: “The company’s valuation is USD 1bn”. **Evaluation** – is a progressing process of examine of physical capability or people characteristics. After something is evaluated, a valuation is often determined. Valuation is an official procedure of examine of the market price. **Appraisals** are only intended as a guide to pricing.

Profits: This is the money that a person or a company gets after they pay for the costs. **Interest**: If I put money in the bank, they give me interest. If I borrow money, I have to give back the money, and interest. **Benefits**: This has two meanings in English. 1: Benefits is money that the government pays to people who are ill or without a job. 2: Benefits is the extra things you get with your job. For example, If I work for a Cinema, I get paid by them, but as part of my benefits, they might give me free film tickets. **Revenue**: This is the money that a company takes from sales. It is different from profits because revenue is all the money a company gets, but they have to pay costs like rent and raw materials. Revenues (**=sales proceeds**) is used for all regular money inflows coming from main company’s business activity. The **gains** from securities (and other financial and investment actions) are not included to Revenues. **Gains** are inflows from peripheral activities. **Income** – mainly, the net of revenues and expenses for the current period. **Retained earnings** = Income (of current period) + net profit of previous periods – paid dividends.

Three types of future cash flow: annuity, perpetuity and variable



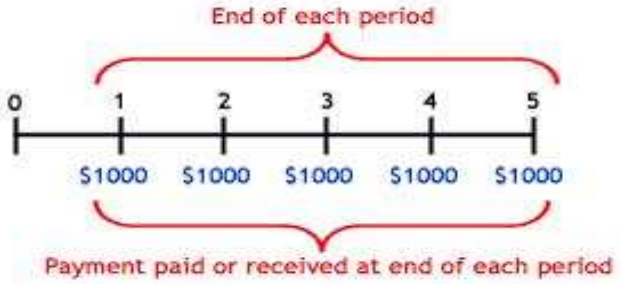


	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Net Cashflow	**$4,300**	**($600)**	**($490)**	**($519)**	**($386)**	**($239)**	**($67)**	**$319**	**$783**	**$1,140**	**$1,807**	**$2,097**
Cash Balance	$4,300	$3,700	$3,210	$2,691	$2,305	$2,066	$1,998	$2,317	$3,100	$4,240	$6,047	$8,144

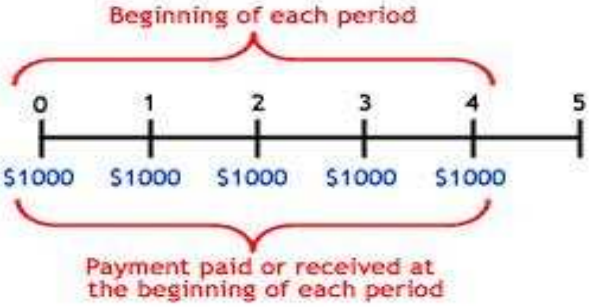
Annuity

An **annuity** is a regular continuing payment with a fixed total annual amount limited for definite numbers of periods.

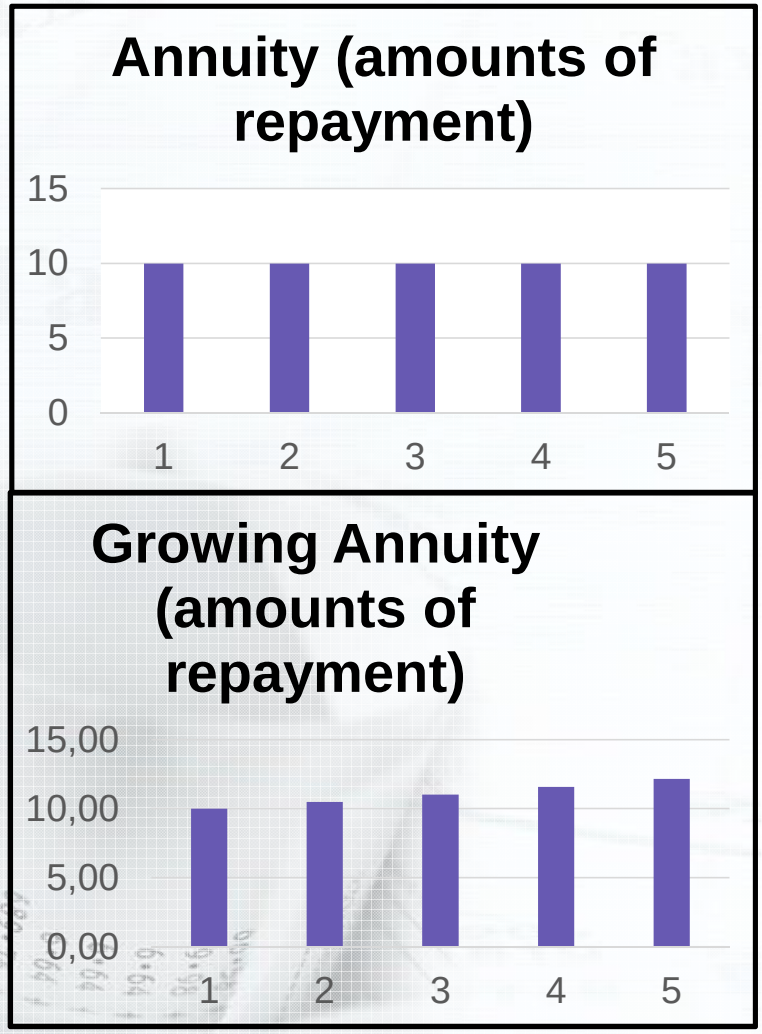
Ordinary Annuity: Payments are required at the end of each period. For example, straight bonds usually pay coupon payments at the end of every six months until the bond's maturity date.

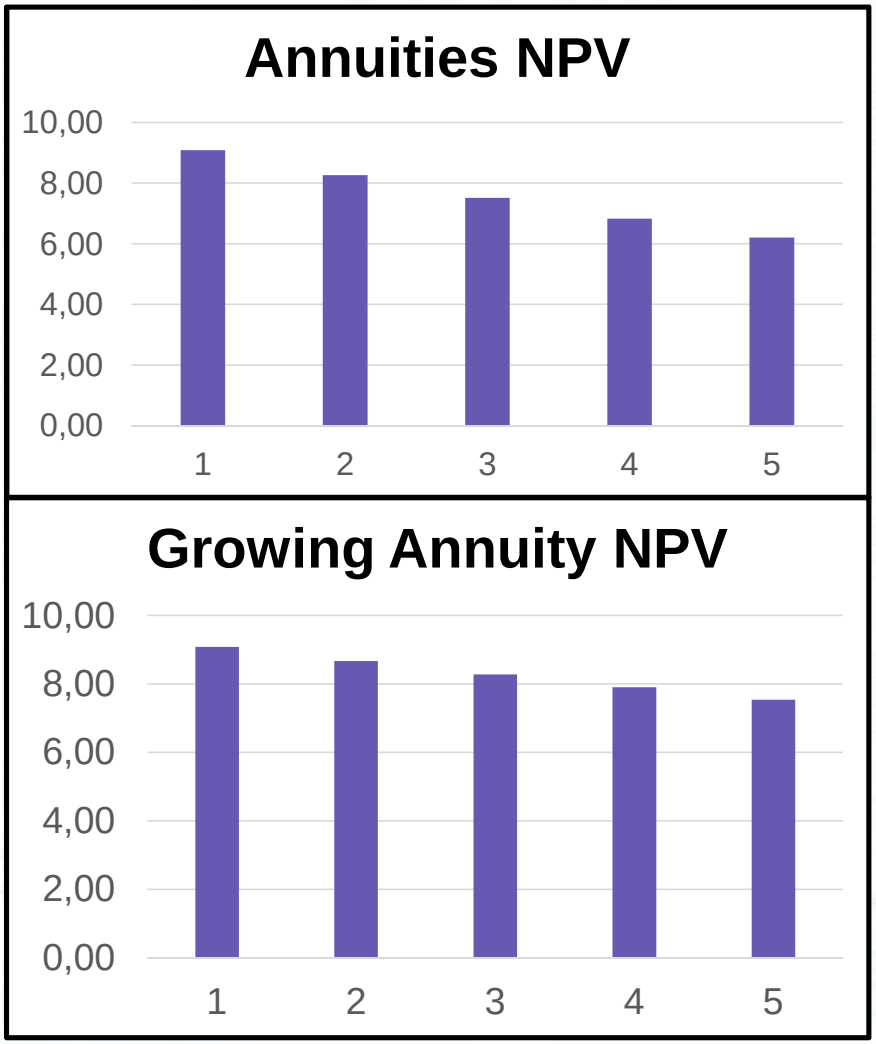


Annuity Due: Payments are required at the beginning of each period. Rent is an example of annuity due. You are usually required to pay rent when you first move in at the beginning of the month and then on the first of each month thereafter.



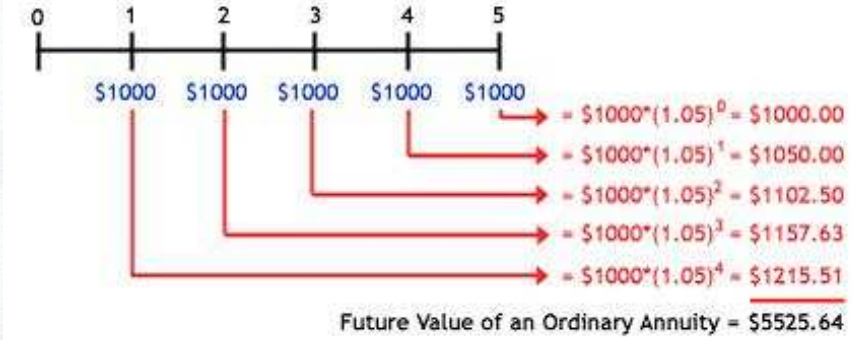
Annuity: Constant & Growing



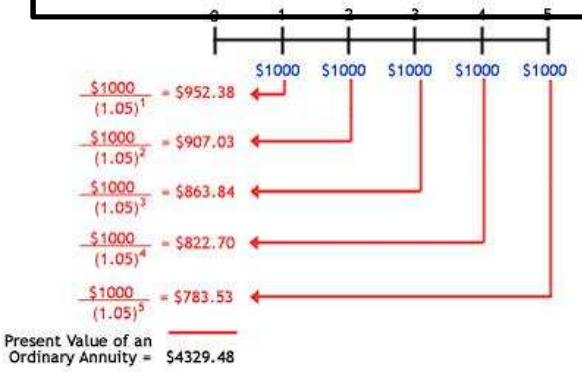


Annuity

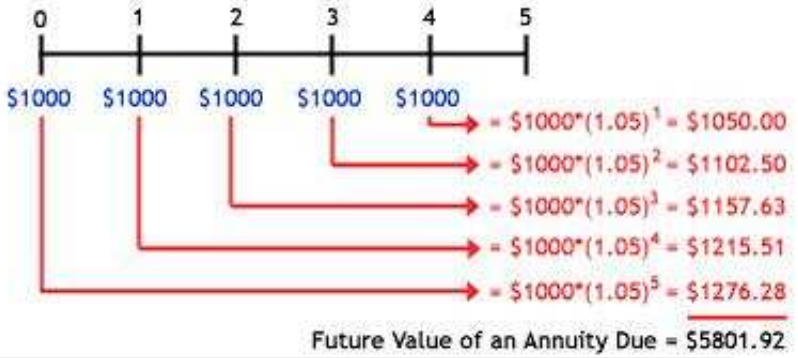
$$FV_{Ordinary\ Annuity} = CF \times \left[\frac{(1+r)^n - 1}{r}\right]$$

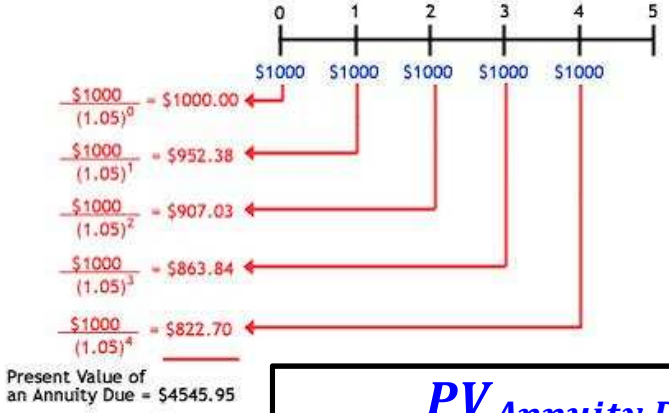


$$PV_{Ordinary\ Annuity} = CF \times \left[\frac{1-(1+r)^{-n}}{r}\right]$$



CF = Cash flow per period
r = interest rate
n = number of payments





With great courtesy to Investopedia.com for the graphs

$$FV_{Annuity\ Due} = CF \times \left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)$$

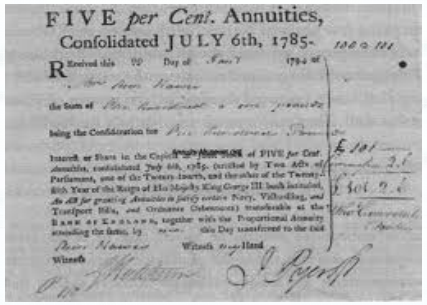
$$PV_{Annuity\ Due} = CF \times \left[\frac{1-(1+r)^{-n}}{r}\right] \times (1+r)$$

Perpetuity

An **perpetuity** is a regular continuing payment with a fixed total annual amount non-limited for indefinite future.

Samples of Perpetuity:

- ***US Treasury bonds*** [T-bonds] issued for 10-30 years periods have up to 360 times of repayments – How to calculate NPV physically? ***It's almost perpetual repayments***.
- Do we have something ***really perpetual*** in the economy? – A super exotic security: ***British Consolidated Annuities*** (officially **termless** bonds issued by the Great Britain Government in 1752-1927 named "***Consols***". The latest yield level was established by Winston Churchill's Government in 1927 in amount of 2,5-4% p.a.).


FIVE *per Cent.* Annuities,
Consolidated JULY 6th, 1785.

Perpetuity

No growth perpetuity

How much do you need to invest today in order to receive yearly 10$ if the rate is 10%?

Discount Rate	***10%***				
Time	**Cash Flow**	**Discount Factor**	**PV**	**Cumulative PV**	
1	10	0,9091	9,09	9,09	
2	10	0,8264	8,26	17,36	
3	10	0,7513	7,51	24,87	
4	10	0,6830	6,83	31,70	
5	10	0,6209	6,21	37,91	
...					
50	10	0,0085	0,09	99,15	
...					
100	10	0,0001	0,00	99,99	
...					
TOTAL PV (The Sum of the Sequence)				100	

The formula for calculation of the Present Value of infinite equal annual payments is:

$$PV_0 = \frac{CF}{r}$$

Perpetuity

Perpetuity With Growth

*How much do you need to invest today in order to receive yearly 10$ **plus 5% Growth** if the rate is 10%?*

Discount Rate	***10%***		***Growth Rate***	***5%***	***Initial Payment***	***$10***
Time	**Cash Flow**	**Discount Factor**	**PV**	**Cumulative PV**		
1	10,00	0,9091	9,09	9,09		
2	10,50	0,8264	8,68	17,77		
3	11,03	0,7513	8,28	26,05		
4	11,58	0,6830	7,91	33,96		
5	12,16	0,6209	7,55	41,51		
...						
27	35,56	0,0763	2,71	143,04		
28	37,33	0,0693	2,59	145,63		
29	39,20	0,0630	2,47	148,10		
30	41,16	0,0573	2,36	150,46		
...						
TOTAL PV (The Sum of the Sequence)				**200**		

The formula for calculation of the Present Value of infinite growing annual payments is:

$$PV_0 = \frac{CF}{r - g}$$

Perpetuity

British Consolidated Annuities

What is a Present value of the Consol with amount 10'000 GBP and interest rate 4% p.a.? Discount rate is 10%.

Discount Rate	*10%*	*Interest rate*	*4%*		*Consol amount*	*£10 000*	
Time	Cash Flow	Discount Factor	PV	Cumulative PV			
1	£400	0,9091	£363,64	£363,64			
2	£400	0,8264	£330,58	£694,21			
3	£400	0,7513	£300,53	£994,74			
4	£400	0,6830	£273,21	£1 267,95			
5	£400	0,6209	£248,37	£1 516,31			
...							
TOTAL PV (The Sum of the Sequence)				£4 000			

The generic formula for calculation of the Present Value of infinite equal annual payments is (if no growth, **g = 0**):

$$PV_0 = \frac{CF}{r-g} \times \left[1 - \frac{(1+g)^n}{(1+r)^n}\right]$$