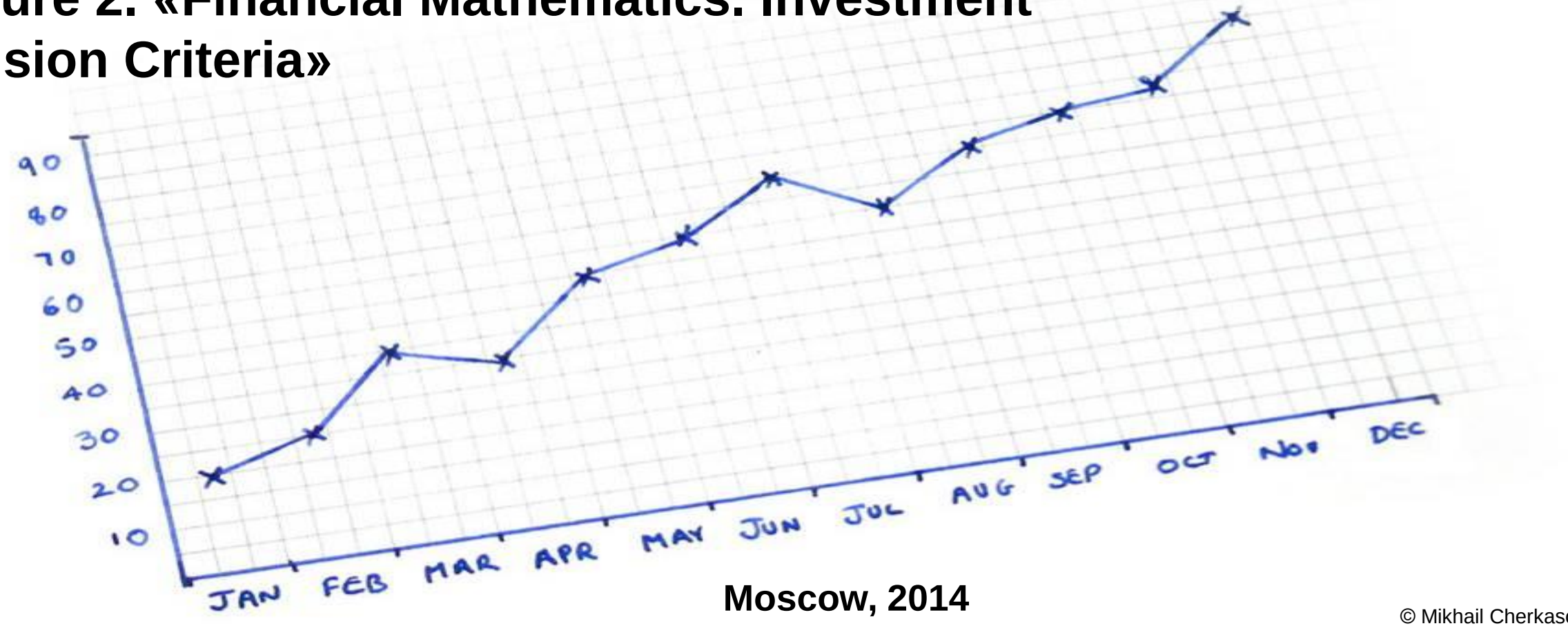
National Research University – Higher School of Economics

# Investment Project Management

## Lecture 2. «Financial Mathematics. Investment Decision Criteria»




Moscow, 2014

# Investment Decision Criteria

The key concept of the financial mathematics is the **NET PRESENT VALUE** or Net Present Worth (NPW). It's a sum of Present Values of time series of future cash flows. The cash flows can be outgoing (out-flows) and incoming (in-flows). NPV can be calculated even for only negative cash flows. It's the most generic and generally recognized measure of comparison of economic projects, enterprises, securities and other assets.

The formula for calculation of the Net Present Value (**NPV**) with given future cash flows for each period **t** (among **n** periods) starting since year 0 ($CF_t$), the investments for each period ($I_t$) and the discount rate (**r**) is:

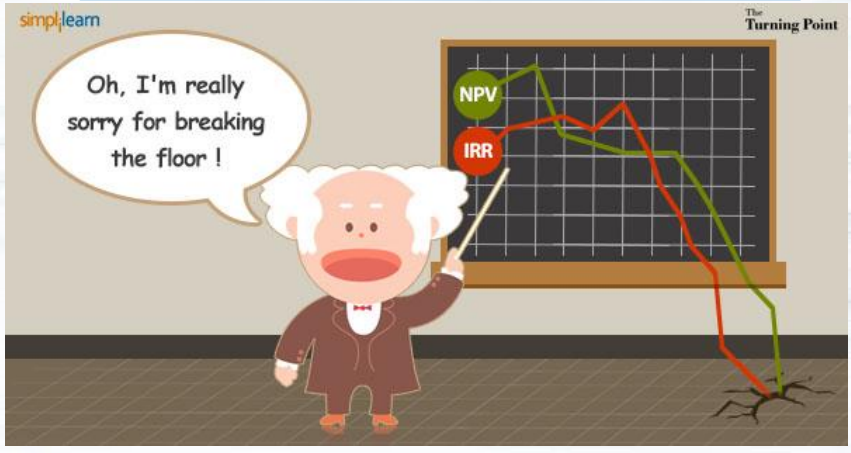
$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}$$

The Discount rate (Hurdle rate):

**r**

The Discount factor:

$$\frac{1}{(1+r)^t}$$



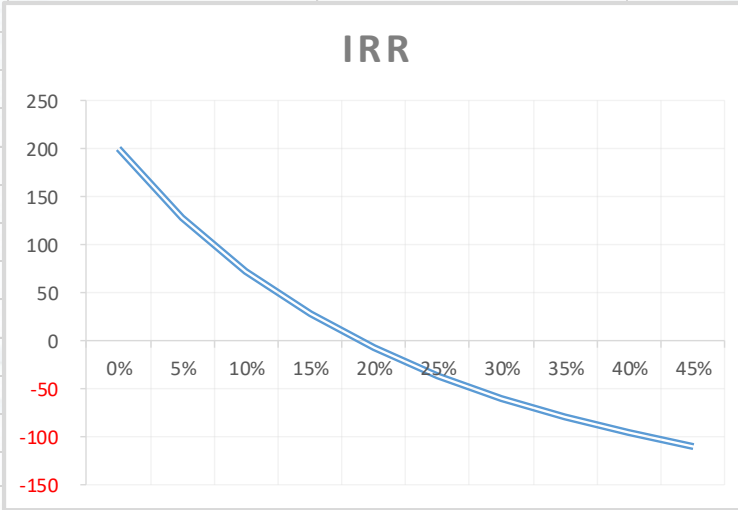
# NPV and IRR

The absolute value of **NET PRESENT VALUE** totally depends on the **DISCOUNT RATE**. The larger is discount rate the less is NPV.

**NPV/IRR Calulator**

| | Discount rates | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | NPV |
|---|---|---|---|---|---|---|---|---|---|
| Number of period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| **Cash Flows** | | **-200** | **0** | **0** | **0** | **400** | **0** | **0** | |
| | 0% | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | |
| **DCF** | | -200 | 0 | 0 | 0 | 400 | 0 | 0 | **200** |
| | 5% | 1,00000 | 0,95238 | 0,90703 | 0,86384 | 0,82270 | 0,78353 | 0,74622 | |
| **DCF** | | -200 | 0 | 0 | 0 | 329 | 0 | 0 | **129** |
| | 10% | 1,00000 | 0,90909 | 0,82645 | 0,75131 | 0,68301 | 0,62092 | 0,56447 | |
| **DCF** | | -200 | 0 | 0 | 0 | 273 | 0 | 0 | **73** |
| | 15% | 1,00000 | 0,86957 | 0,75614 | 0,65752 | 0,57175 | 0,49718 | 0,43233 | |
| **DCF** | | -200 | 0 | 0 | 0 | 229 | 0 | 0 | **29** |
| | 20% | 1,00000 | 0,83333 | 0,69444 | 0,57870 | 0,48225 | 0,40188 | 0,33490 | |
| **DCF** | | -200 | 0 | 0 | 0 | 193 | 0 | 0 | **-7** |
| | 25% | 1,00000 | 0,80000 | 0,64000 | 0,51200 | 0,40960 | 0,32768 | 0,26214 | |
| **DCF** | | -200 | 0 | 0 | 0 | 164 | 0 | 0 | **-36** |
| | 30% | 1,00000 | 0,76923 | 0,59172 | 0,45517 | 0,35013 | 0,26933 | 0,20718 | |
| **DCF** | | -200 | 0 | 0 | 0 | 140 | 0 | 0 | **-60** |
| | 35% | 1,00000 | 0,74074 | 0,54870 | 0,40644 | 0,30107 | 0,22301 | 0,16520 | |
| **DCF** | | -200 | 0 | 0 | 0 | 120 | 0 | 0 | **-80** |
| | 40% | 1,00000 | 0,71429 | 0,51020 | 0,36443 | 0,26031 | 0,18593 | 0,13281 | |
| **DCF** | | -200 | 0 | 0 | 0 | 104 | 0 | 0 | **-96** |
| | 45% | 1,00000 | 0,68966 | 0,47562 | 0,32802 | 0,22622 | 0,15601 | 0,10759 | |
| **DCF** | | -200 | 0 | 0 | 0 | 90 | 0 | 0 | **-110** |

| *Discount Rates input* | *Cash flows input* | *NPVs* |
|---|---|---|
| 0,00% | **-200** | 200 |
| 5,00% | **0** | 129 |
| 10,00% | **0** | 73 |
| 15,00% | **0** | 29 |
| 20,00% | **400** | -7 |
| 25,00% | **0** | -36 |
| 30,00% | | -60 |
| 35,00% | | -80 |
| 40,00% | | -96 |
| 45,00% | | -110 |



# NPV and IRR

## NPV calculation:

| | | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| Period number | | 1 | 2 | 3 | 4 | 5 | 6 | |
| Cash flow for the period (after tax) | | -5 000 | 3 845 | -300 | 6 920 | 8 691 | 10 477 | 24 633 |
| Discount rate (Hurdle rate) | *12%* | 1,0000 | 1,1200 | 1,2544 | 1,4049 | 1,5735 | 1,7623 | |
| Discount factor | | 1,0000 | 0,8929 | 0,7972 | 0,7118 | 0,6355 | 0,5674 | |
| PV (Present Value) after tax | | -5 000,00 | 3 433,04 | -239,16 | 4 925,64 | 5 523,25 | 5 944,97 | 14 588 |

NPV

# NPV and IRR

## NPV calculation:

| | | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| **Period number** | | **1** | **2** | **3** | **4** | **5** | **6** | |
| **Discount rate (Hurdle rate)** | ***12%*** | 1,0000 | 1,1200 | 1,2544 | 1,4049 | 1,5735 | 1,7623 | |
| **Discount factor** | | 1,0000 | 0,8929 | 0,7972 | 0,7118 | 0,6355 | 0,5674 | |
| Discounted cash outflow for the period | | -5 000 | -11 746 | -9 008 | -12 869 | -13 542 | -13 915 | **-66 080** |
| Cumulative Discounted cash outflow | | -5 000 | -16 746 | -25 754 | -38 623 | -52 165 | -66 080 | -66 080 |
| Discounted cash inflow for the period | | 0 | 15 179 | 8 769 | 17 795 | 19 066 | 19 860 | **80 668** |
| Cumulative Discounted cash inflow | | 0 | 15 179 | 23 948 | 41 742 | 60 808 | 80 668 | 80 668 |
| | | | | | | | | **14 588** |

# NPV and IRR

## NPV calculation:

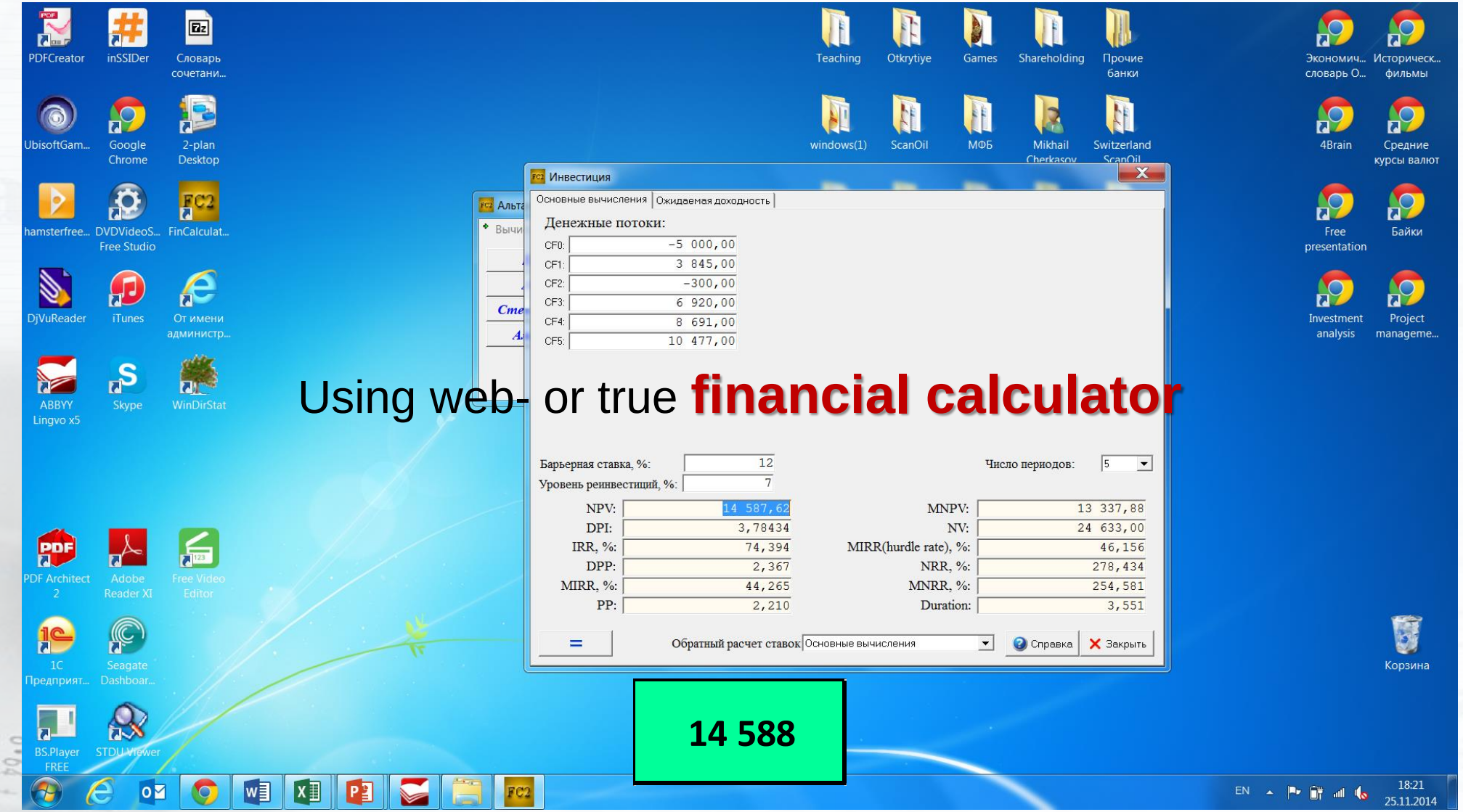
| | | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| Period number | | 1 | 2 | 3 | 4 | 5 | 6 | |
| Cash flow for the period (after tax) | | -5 000 | 3 845 | -300 | 6 920 | 8 691 | 10 477 | 24 633 |
| Discount rate (Hurdle rate) | *12%* | | | | | | | |

Using Excel financial formula "**NPV**" («**ЧПС**»)

**14 588**

# NPV and IRR

## NPV calculation:




Using web- or true **financial calculator**

**14 588**

Not bad web-calculator - *http://investment-analysis.ru/*

# NPV and IRR

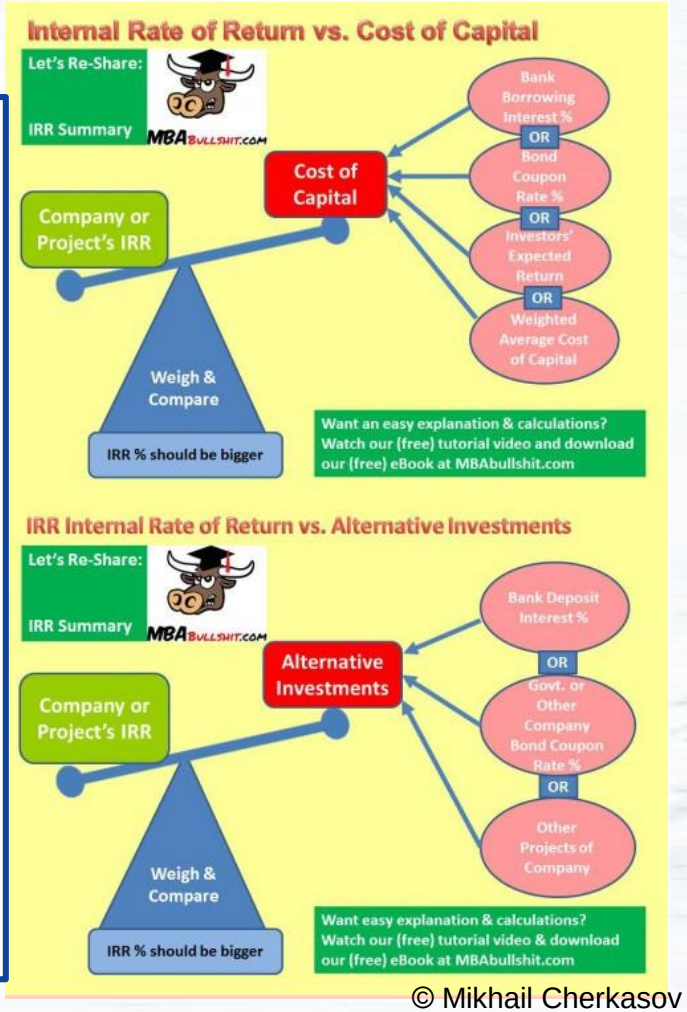
**Internal Rate of Return** (**IRR**) or Economic Rate of Return or Discounted Cash Flow Rate of Return or Annualized Effected Compounded Return Rate. IRR reflects the discount rate when future cash flows' NPV is equal to zero.

The formula for calculation of the Internal Rate of Return (**IRR**) with given future cash flows for each period **t** (among **n** periods) starting since year 0 ($CF_t$), the investments for each period ($I_t$) and the discount rate (**r**) is:

$$IRR = r, when \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} = \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}$$

or:

$$\sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} = \sum_{t=0}^{n} \frac{I_t}{(1+IRR)^t}$$



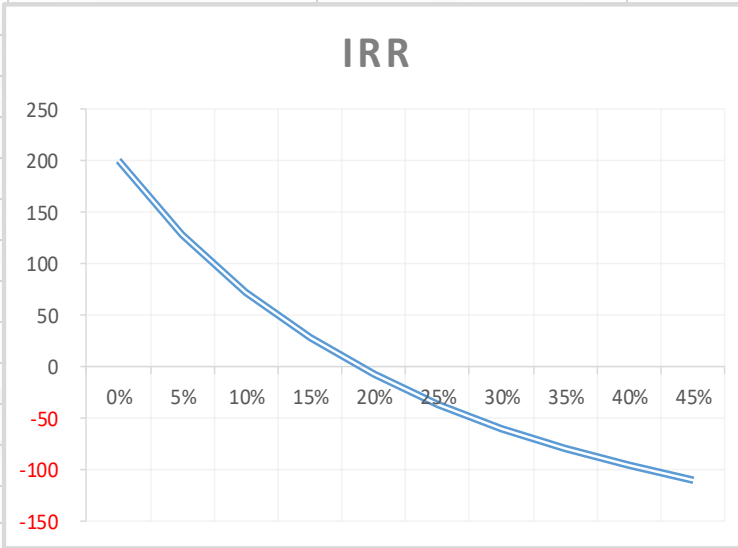
# NPV and IRR

## Internal Rate of Return (IRR) calculation:

**NPV/IRR Calulator**

| | Discount rates | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | NPV |
|---|---|---|---|---|---|---|---|---|---|
| Number of period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| **Cash Flows** | | **-200** | **0** | **0** | **0** | **400** | **0** | **0** | |
| | 0% | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | 1,00000 | |
| **DCF** | | -200 | 0 | 0 | 0 | 400 | 0 | 0 | **200** |
| | 5% | 1,00000 | 0,95238 | 0,90703 | 0,86384 | 0,82270 | 0,78353 | 0,74622 | |
| **DCF** | | -200 | 0 | 0 | 0 | 329 | 0 | 0 | **129** |
| | 10% | 1,00000 | 0,90909 | 0,82645 | 0,75131 | 0,68301 | 0,62092 | 0,56447 | |
| **DCF** | | -200 | 0 | 0 | 0 | 273 | 0 | 0 | **73** |
| | 15% | 1,00000 | 0,86957 | 0,75614 | 0,65752 | 0,57175 | 0,49718 | 0,43233 | |
| **DCF** | | -200 | 0 | 0 | 0 | 229 | 0 | 0 | **29** |
| | 20% | 1,00000 | 0,83333 | 0,69444 | 0,57870 | 0,48225 | 0,40188 | 0,33490 | |
| **DCF** | | -200 | 0 | 0 | 0 | 193 | 0 | 0 | **-7** |
| | 25% | 1,00000 | 0,80000 | 0,64000 | 0,51200 | 0,40960 | 0,32768 | 0,26214 | |
| **DCF** | | -200 | 0 | 0 | 0 | 164 | 0 | 0 | **-36** |
| | 30% | 1,00000 | 0,76923 | 0,59172 | 0,45517 | 0,35013 | 0,26933 | 0,20718 | |
| **DCF** | | -200 | 0 | 0 | 0 | 140 | 0 | 0 | **-60** |
| | 35% | 1,00000 | 0,74074 | 0,54870 | 0,40644 | 0,30107 | 0,22301 | 0,16520 | |
| **DCF** | | -200 | 0 | 0 | 0 | 120 | 0 | 0 | **-80** |
| | 40% | 1,00000 | 0,71429 | 0,51020 | 0,36443 | 0,26031 | 0,18593 | 0,13281 | |
| **DCF** | | -200 | 0 | 0 | 0 | 104 | 0 | 0 | **-96** |
| | 45% | 1,00000 | 0,68966 | 0,47562 | 0,32802 | 0,22622 | 0,15601 | 0,10759 | |
| **DCF** | | -200 | 0 | 0 | 0 | 90 | 0 | 0 | **-110** |

| *Discount Rates input* | *Cash flows input* | *NPVs* |
|---|---|---|
| 0,00% | **-200** | 200 |
| 5,00% | **0** | 129 |
| 10,00% | **0** | 73 |
| 15,00% | **0** | 29 |
| 20,00% | **400** | -7 |
| 25,00% | **0** | -36 |
| 30,00% | | -60 |
| 35,00% | | -80 |
| 40,00% | | -96 |
| 45,00% | | -110 |



# NPV and IRR

## Internal Rate of Return (IRR) calculation:

| IRR | Rate | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| **IRR (Internal Rate of Return)** *calculated with Excel* | | | | | | | | 74,39% |
| | | | | | | Excel "IRR" or «ВСД» | | |
| *Iterations:* | | | | | | | | |
| Discount factor | *70%* | 1,0000 | 0,5882 | 0,3460 | 0,2035 | 0,1197 | 0,0704 | |
| NPV | | -5 000,00 | -2 738,24 | -2 842,04 | -1 433,50 | -392,93 | 344,96 | |
| Discount factor | *74%* | 1,0000 | 0,5747 | 0,3303 | 0,1898 | 0,1091 | 0,0627 | |
| NPV | | -5 000,00 | -2 790,23 | -2 889,32 | -1 575,70 | -627,57 | 29,32 | |
| Discount factor | *74,39%* | 1,0000 | 0,5734 | 0,3288 | 0,1885 | 0,1081 | 0,0620 | |
| NPV | | -5 000,00 | -2 795,22 | -2 893,86 | -1 589,12 | -649,51 | -0,00 | |
| Discount factor | *75%* | 1,0000 | 0,5714 | 0,3265 | 0,1866 | 0,1066 | 0,0609 | |
| NPV | | -5 000,00 | -2 802,86 | -2 900,82 | -1 609,59 | -682,94 | -44,61 | |

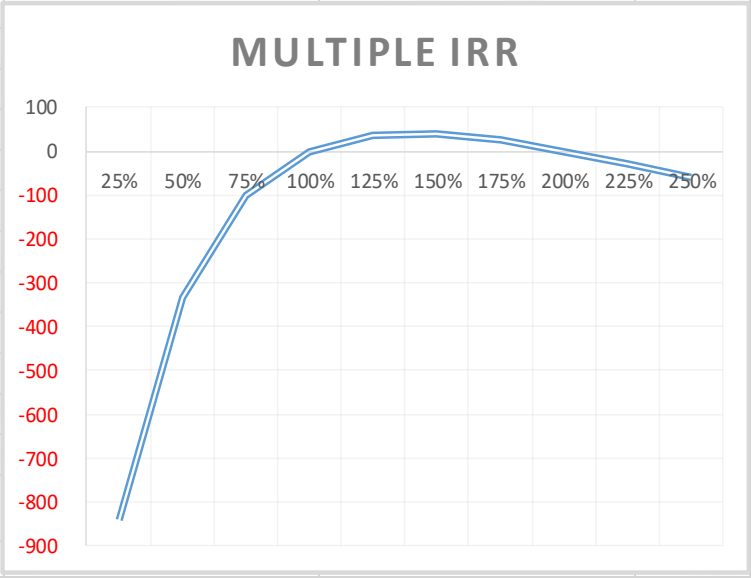
# NPV and IRR

## Multiple **Internal Rate of Return** (**IRR**)

**Multiple IRR**

| | Discount rates | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | NPV |
|---|---|---|---|---|---|---|---|---|---|
| Number of period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| **Cash Flows** | | **-1 000** | **5 000** | **-6 000** | **0** | **0** | **0** | **0** | |
| | 25% | 1,00000 | 0,80000 | 0,64000 | 0,51200 | 0,40960 | 0,32768 | 0,26214 | |
| **DCF** | | -1 000 | 4 000 | -3 840 | 0 | 0 | 0 | 0 | **-840** |
| | 50% | 1,00000 | 0,66667 | 0,44444 | 0,29630 | 0,19753 | 0,13169 | 0,08779 | |
| **DCF** | | -1 000 | 3 333 | -2 667 | 0 | 0 | 0 | 0 | **-333** |
| | 75% | 1,00000 | 0,57143 | 0,32653 | 0,18659 | 0,10662 | 0,06093 | 0,03482 | |
| **DCF** | | -1 000 | 2 857 | -1 959 | 0 | 0 | 0 | 0 | **-102** |
| | 100% | 1,00000 | 0,50000 | 0,25000 | 0,12500 | 0,06250 | 0,03125 | 0,01563 | |
| **DCF** | | -1 000 | 2 500 | -1 500 | 0 | 0 | 0 | 0 | **0** |
| | 125% | 1,00000 | 0,44444 | 0,19753 | 0,08779 | 0,03902 | 0,01734 | 0,00771 | |
| **DCF** | | -1 000 | 2 222 | -1 185 | 0 | 0 | 0 | 0 | **37** |
| | 150% | 1,00000 | 0,40000 | 0,16000 | 0,06400 | 0,02560 | 0,01024 | 0,00410 | |
| **DCF** | | -1 000 | 2 000 | -960 | 0 | 0 | 0 | 0 | **40** |
| | 175% | 1,00000 | 0,36364 | 0,13223 | 0,04808 | 0,01749 | 0,00636 | 0,00231 | |
| **DCF** | | -1 000 | 1 818 | -793 | 0 | 0 | 0 | 0 | **25** |
| | 200% | 1,00000 | 0,33333 | 0,11111 | 0,03704 | 0,01235 | 0,00412 | 0,00137 | |
| **DCF** | | -1 000 | 1 667 | -667 | 0 | 0 | 0 | 0 | **-0** |
| | 225% | 1,00000 | 0,30769 | 0,09467 | 0,02913 | 0,00896 | 0,00276 | 0,00085 | |
| **DCF** | | -1 000 | 1 538 | -568 | 0 | 0 | 0 | 0 | **-30** |
| | 250% | 1,00000 | 0,28571 | 0,08163 | 0,02332 | 0,00666 | 0,00190 | 0,00054 | |
| **DCF** | | -1 000 | 1 429 | -490 | 0 | 0 | 0 | 0 | **-61** |
| **Excel** | **100%** | | | | | | | | |

| *Discount Rates input* | *Cash flows input* | *NPVs* |
|---|---|---|
| 25,00% | **-1 000** | -840 |
| 50,00% | **5 000** | -333 |
| 75,00% | **-6 000** | -102 |
| 100,00% | | 0 |
| 125,00% | | 37 |
| 150,00% | | 40 |
| 175,00% | | 25 |
| 200,00% | | -0 |
| 225,00% | | -30 |
| 250,00% | | -61 |

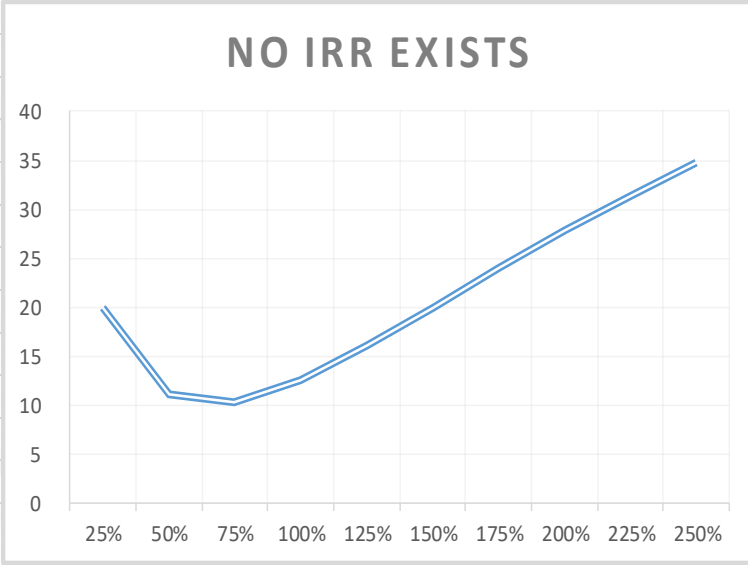


# NPV and IRR

## No Internal Rate of Return (IRR)

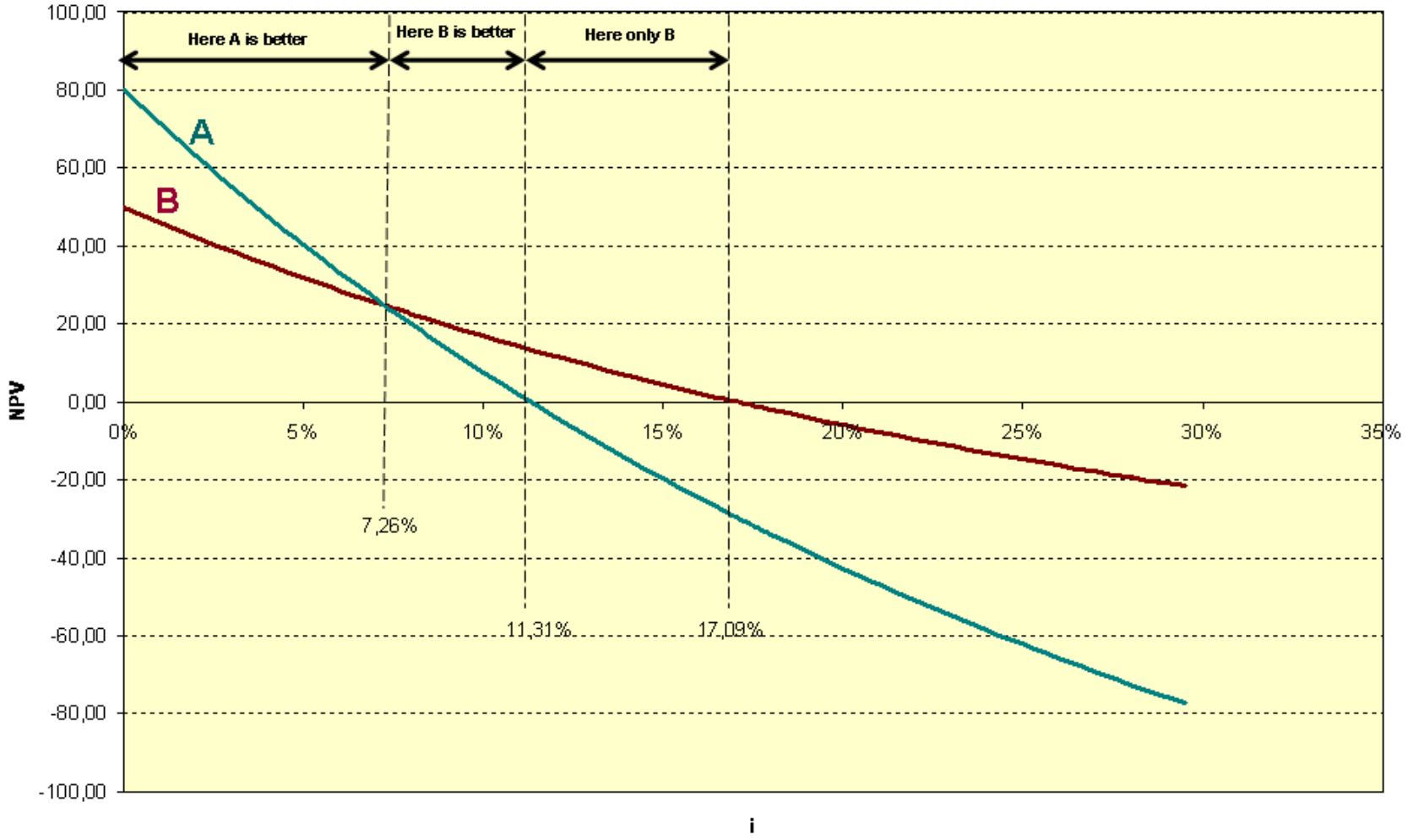
| No IRR | Discount rates | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | NPV |
|---|---|---|---|---|---|---|---|---|---|
| Number of period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| Cash Flows | | 100 | -300 | 250 | 0 | 0 | 0 | 0 | |
| | 25% | 1,00000 | 0,80000 | 0,64000 | 0,51200 | 0,40960 | 0,32768 | 0,26214 | |
| DCF | | 100 | -240 | 160 | 0 | 0 | 0 | 0 | 20 |
| | 50% | 1,00000 | 0,66667 | 0,44444 | 0,29630 | 0,19753 | 0,13169 | 0,08779 | |
| DCF | | 100 | -200 | 111 | 0 | 0 | 0 | 0 | 11 |
| | 75% | 1,00000 | 0,57143 | 0,32653 | 0,18659 | 0,10662 | 0,06093 | 0,03482 | |
| DCF | | 100 | -171 | 82 | 0 | 0 | 0 | 0 | 10 |
| | 100% | 1,00000 | 0,50000 | 0,25000 | 0,12500 | 0,06250 | 0,03125 | 0,01563 | |
| DCF | | 100 | -150 | 63 | 0 | 0 | 0 | 0 | 13 |
| | 125% | 1,00000 | 0,44444 | 0,19753 | 0,08779 | 0,03902 | 0,01734 | 0,00771 | |
| DCF | | 100 | -133 | 49 | 0 | 0 | 0 | 0 | 16 |
| | 150% | 1,00000 | 0,40000 | 0,16000 | 0,06400 | 0,02560 | 0,01024 | 0,00410 | |
| DCF | | 100 | -120 | 40 | 0 | 0 | 0 | 0 | 20 |
| | 175% | 1,00000 | 0,36364 | 0,13223 | 0,04808 | 0,01749 | 0,00636 | 0,00231 | |
| DCF | | 100 | -109 | 33 | 0 | 0 | 0 | 0 | 24 |
| | 200% | 1,00000 | 0,33333 | 0,11111 | 0,03704 | 0,01235 | 0,00412 | 0,00137 | |
| DCF | | 100 | -100 | 28 | 0 | 0 | 0 | 0 | 28 |
| | 225% | 1,00000 | 0,30769 | 0,09467 | 0,02913 | 0,00896 | 0,00276 | 0,00085 | |
| DCF | | 100 | -92 | 24 | 0 | 0 | 0 | 0 | 31 |
| | 250% | 1,00000 | 0,28571 | 0,08163 | 0,02332 | 0,00666 | 0,00190 | 0,00054 | |
| DCF | | 100 | -86 | 20 | 0 | 0 | 0 | 0 | 35 |
| Excel | #ЧИСЛО! | | | | | | | | |

| *Discount Rates input* | *Cash flows input* | *NPVs* |
|---|---|---|
| 25,00% | 100 | 20 |
| 50,00% | -300 | 11 |
| 75,00% | 250 | 10 |
| 100,00% | | 13 |
| 125,00% | | 16 |
| 150,00% | | 20 |
| 175,00% | | 24 |
| 200,00% | | 28 |
| 225,00% | | 31 |
| 250,00% | | 35 |



# NPV and IRR

The Project is acceptable when the **Net Present Value** of forecasted cash flows is > 0 and the **Internal Rate of Return** (**IRR**) is > Agreed Discount rate:



# NPV and IRR

We choose among two Projects that one which has larger **Net Present Value**. The comparison of Project's **Internal Rate of Return** (**IRR**) is secondary.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Project A** | | -200 | 80 | 80 | 80 | 80 | **38,38** | **21,86%** |
| **Project B** | | -200 | 0 | 0 | 0 | 400 | **48,40** | **18,92%** |
| **Discount rate** | *12%* | | | | | | | |

**Larger NPV has a priority over "IRR"**

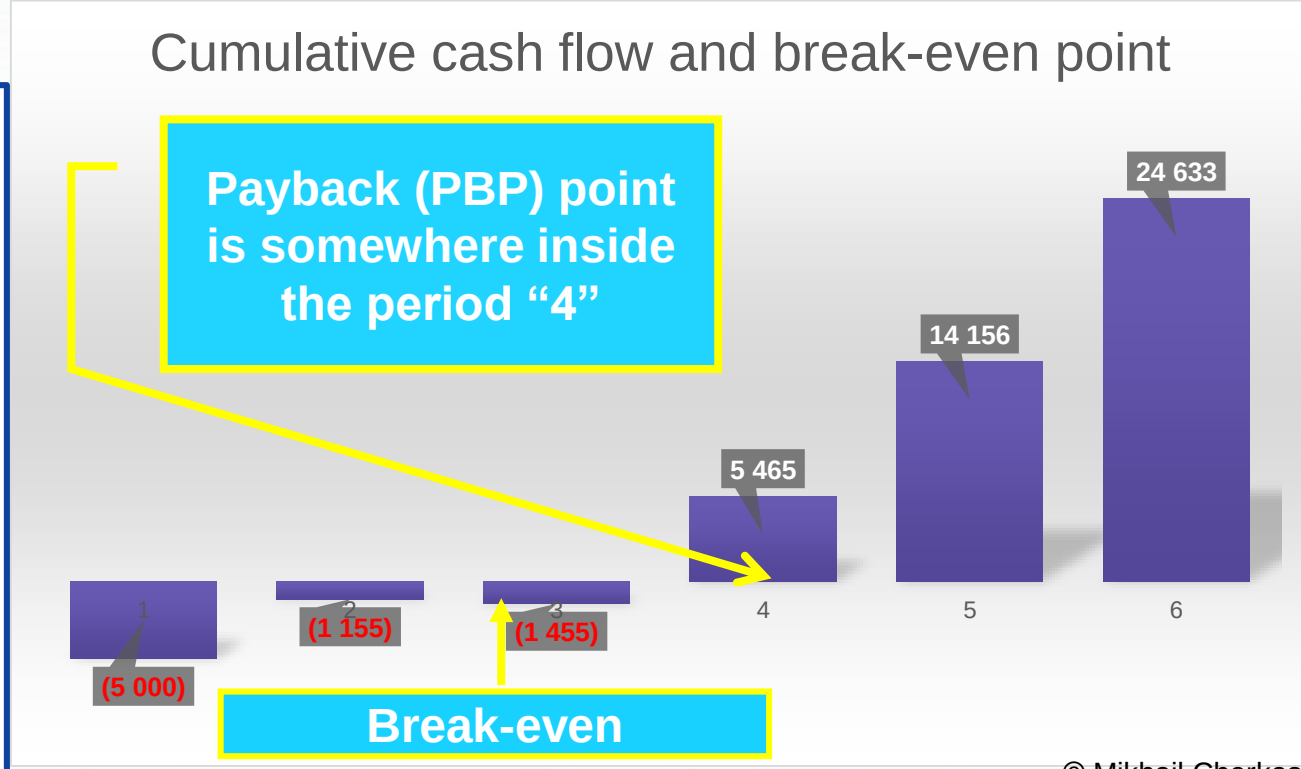
# Payback period

The **Break-even point** is a timeline point where the Project's revenues and expenses are equal. The revenues/outlay balance can be measured in money or in produced units.

The **Payback Period** (**PBP**, **PP**) is a time needed to reach a point where cumulative inflows reach the total amount of the initial investments. . Or the period of time required for the Project's operating inflows *[absolute value]* to reach the Project's initial investments (Project Outlay) *[absolute value]*. Years till the first revenue are not included.

The formula for calculation of the Payback period (**PP**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 ($CF_t$) and the investments for each period ($I_t$) is:

$$PP = n, when \sum_{t=0}^{n} CF_t = \sum_{t=0}^{n} I_t$$



# Payback period

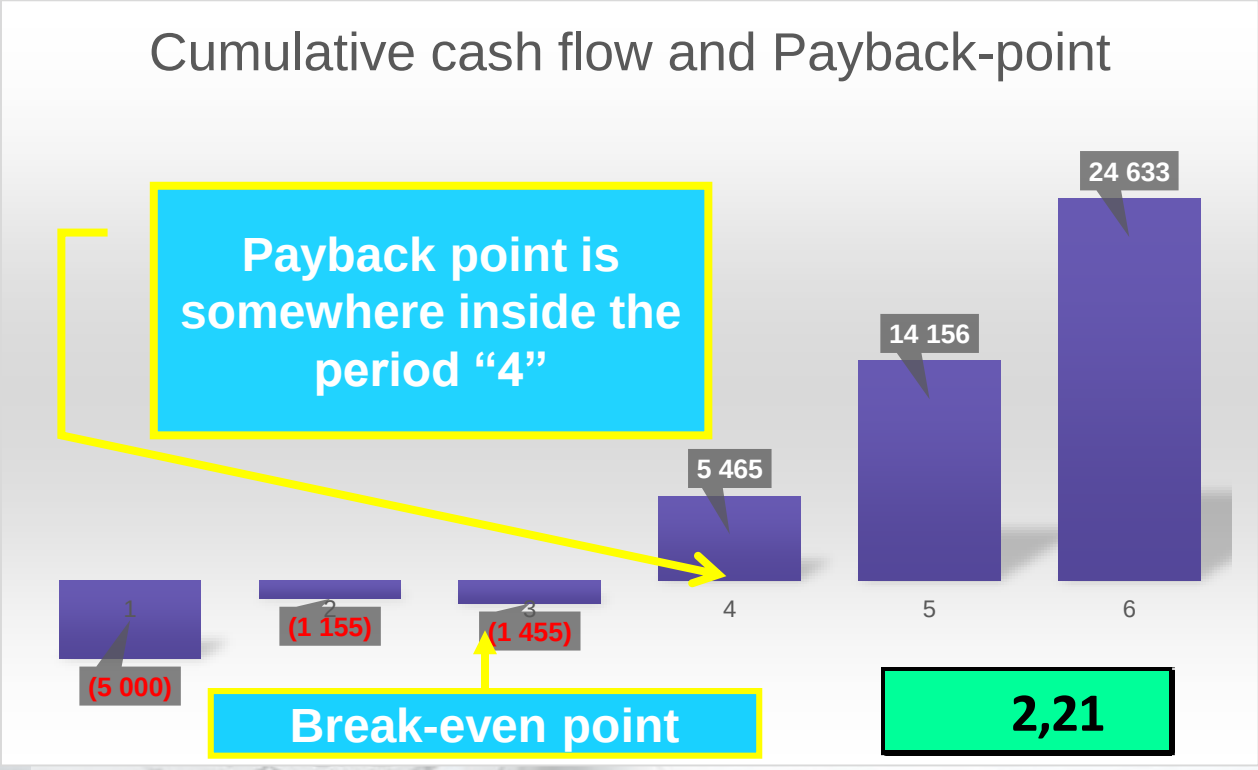
## The **Payback Period** (**PBP**) calculation:

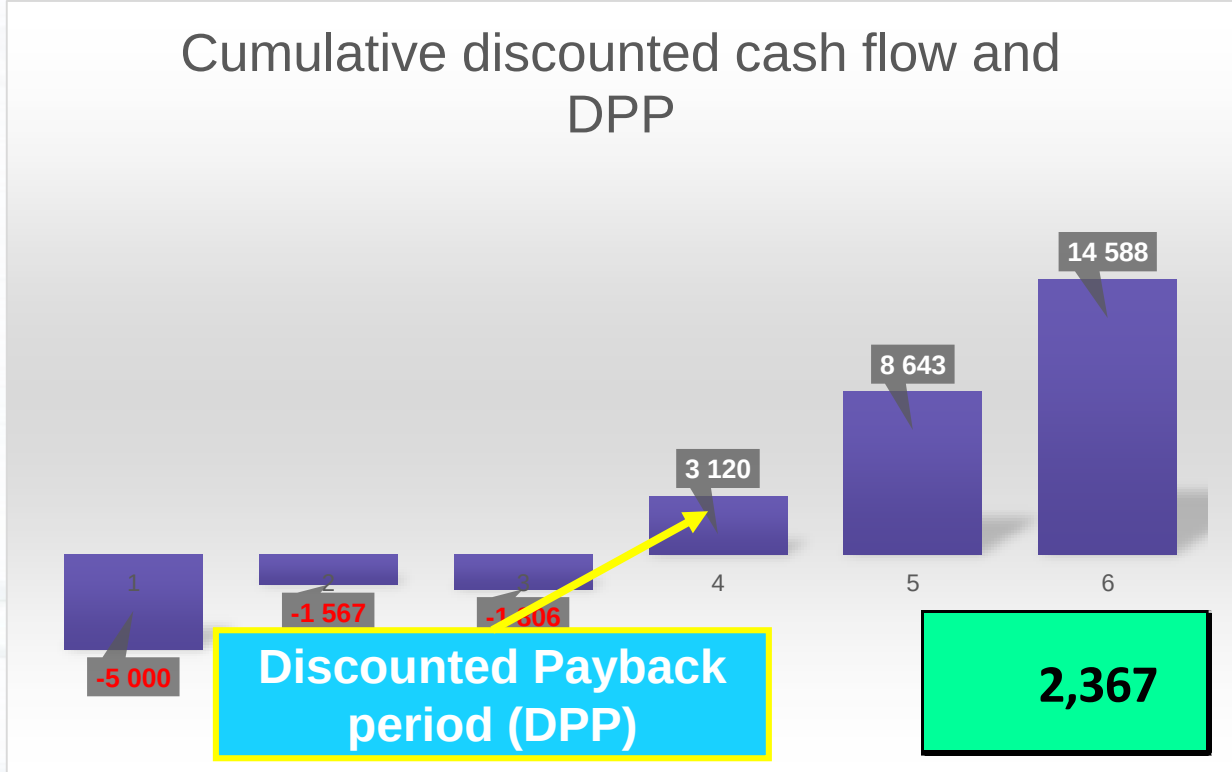
| PP | | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| **NCF (Net Cash Flow) after tax** | | -5 000 | 3 845 | -300 | 6 920 | 8 691 | 10 477 | |
| **Cumulative cash flow** | | -5 000 | -1 155 | -1 455 | 5 465 | 14 156 | 24 633 | |
| **PP (Payback Period).** Revenues come evenly inside the year | | 0 | 0 | 0 | 1 | 1 | 1 | |
| | The function of comparison (IF or "ЕСЛИ") for cumulative CF: ">" or "<" than "0" | | | | | | | |
| **Exact Payback date** | | 0,00 | 0,00 | 0,00 | 0,79 | 0,00 | 0,00 | |
| =ЕСЛИ(И(Previous cell=0;Current cell=1);Exceeding 0 cumulative cash flow/Total cash flow for certain period;0) | | | | | | | | |
| =IF(AND(Previous cell=0;Current cell=1);Exceeding 0 cumulative cash flow/Total cash flow for certain period;0) | | | | | | | | |
| Exact Pay Back Period = СУММ(Years with positive cumulative cash)-СУММ(of row "Exact payment date") | | | | | | | | **2,21** |

**Year0** is out of consideration. The PBP is calculated since the first revenue comes.

# Discounted Payback period

The **Discounted Payback Period** (**DPP**, **Present Value Payback**) is a period of time required for the Project's **discounted** revenues *[absolute value]* to reach the Project's **discounted** initial investments (Project Outlay) *[absolute value]*. Years till the first revenue are not included.



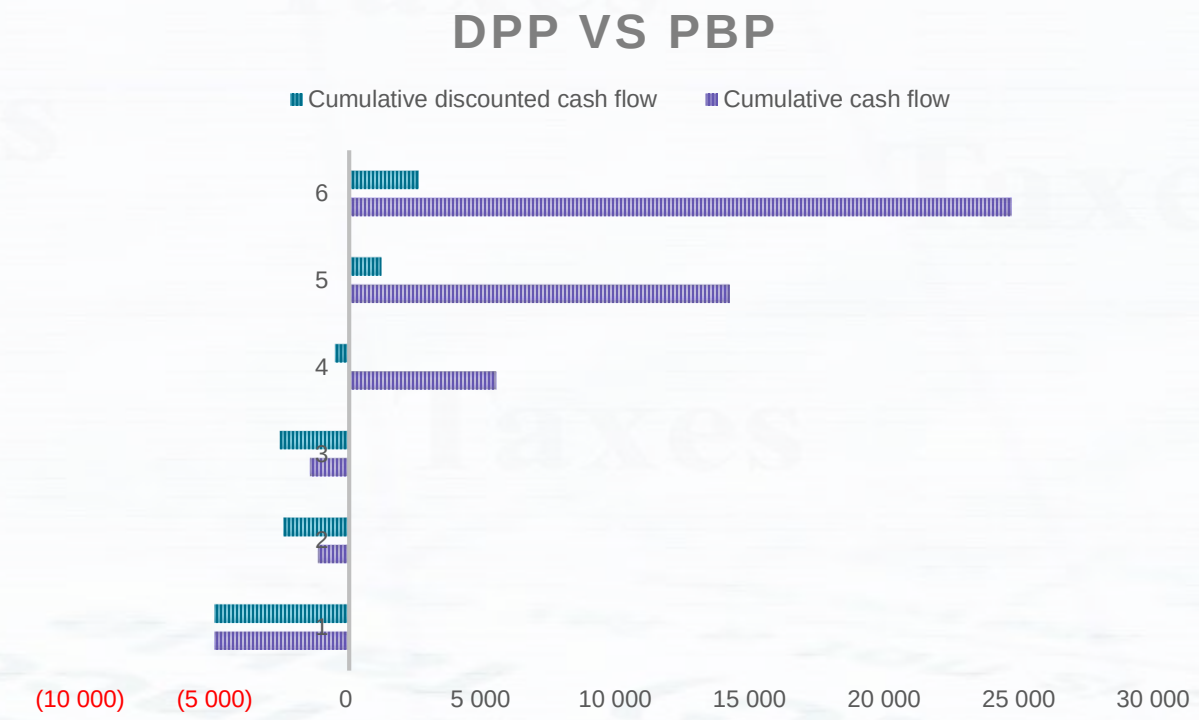
# Discounted Payback period

The **Discounted Payback Period** (**DPP**, **Present Value Payback**) is always longer than **the Payback Period** (**PBP**) because future cash inflows for DPP are discounted (in distinct of PP non-discounted cash inflows). DPP is calculated as:

The formula for calculation of the Discounted Payback period (**DPP**) with given future cash flows for each period (among **n** periods) starting since year 1 ($CF_t$), the investments for each period ($I_t$) and the discount rate (**r**) is:

$$DPP = n, when \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}$$

$$= \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}$$




*For this graph – applied discount rate is 50%.*

# Discounted Profitability Index

The **Discounted Profitability Index** (**DPI**) or PVI (Present Value Index, PV-index), Benefit to Cost Ratio (BCR), Profit-Investment Ratio (PIR), NPWI (Net Present Worth Index) is a quotient of summarized discounted cash inflows to summarized discounted cash outflows and demonstrates the ratio of payoff to investment injected to the Project.

The formula for calculation of the Discounted Profitability Index (**DPI**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 ($CF_t$), the investments for each period ($I_t$) and the discount rate (**r**) is:

$$DPI = \frac{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}{\sum_{t=0}^{n} \frac{I_t}{(1+r)^t}} = \frac{NPV + \sum_{t=0}^{n} \frac{I_t}{(1+r)^t}}{\sum_{t=0}^{n} \frac{I_t}{(1+r)^t}}$$

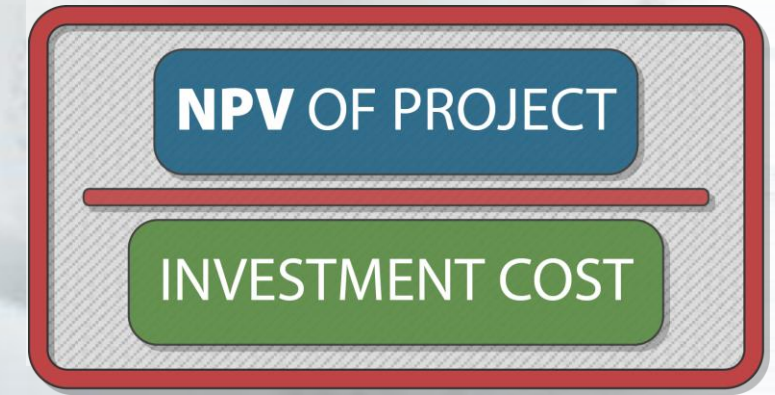
$$DPI = 1 + \frac{NPV}{\sum_{t=0}^{n} \frac{I_t}{(1+r)^t}}$$

# Discounted Profitability Index

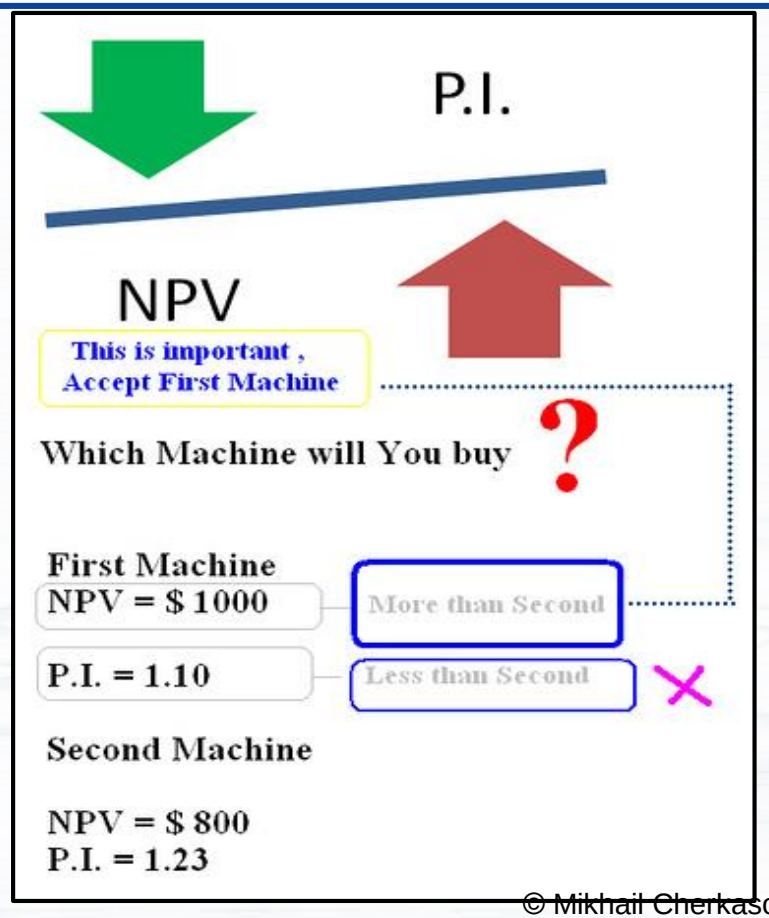
As far as the **Discounted Profitability Index** (**DPI**) is calculated according to following formula:

$$DPI = 1 + \frac{NPV}{Discounted\ Investment}$$

the criteria for the Projects selection is:
DPI > 1 – the project is acceptable;
DPI < 1 – to be refused.
It's a linear consequence of NPV-based selection:
NPV > 0 – to be performed;
NPV < 0 – to be expelled.




*DPI is lower graded criteria than NPV*



# Annualized Net Present Value

The **Annualized Net Present Value** (**ANPV**) or NUS (Net Uniform Series), EAC (Equivalent annual cost), EAA (Equivalent Annual Annuity) is the most efficient technique to convert Net Present Values (NPVs) of Projects with unequal duration into an ANPV for each specific project, which can then be compared.

The formula for calculation of the Annualized Net Present Value (**ANPV**) with given future cash flows for each period **t** (among **n** periods) starting since year 1 ($CF_t$), the investments for each period ($I_t$) and the discount rate (**r**) is:

$$ANPV = \frac{NPV}{\sum_{t=1}^{n} \frac{1}{(1+\frac{r}{100})^t}}$$

**ANPV** calculates the amount of annual cash in-flow which gives the same NPV as given unequal cash flows. ANPV is mainly used for the comparison of the Projects with **unequal maturities**.

# Annualized Net Present Value

The actuality of **Annualized Net Present Value** (**ANPV**) or NUS (Net Uniform Series), EAC (Equivalent annual cost), EAA (Equivalent Annual Annuity) might be illustrated by following sample when there are several Projects with unequal duration:

| | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 | NPV | ANPV/EAA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Project A** | -100 | 115 | | | | | | | | 4,13 ₽ | 4,55 ₽ |
| **Project B** | -150 | 30 | 30 | 20 | 20 | 20 | 10 | 30 | 90 | 5,63 ₽ | 1,06 ₽ |
| ***Discount rate*** | ***10%*** | | | | | | | | | | |

**ANPV** calculates the amount of annual **equal** cash in-flow which gives the same NPV as given **unequal** cash flows. ANPV is mainly used for the comparison of the Projects with **unequal maturities**. Indeed, when the Investor considers 2 alternative projects, one of which gives rather less NPV but the maturities have huge difference, the Investor most likely chooses the shortest one because of unpredictable risks for the longer project. But in order to calculate efficiency precisely in this case ANPV/EAA looks as perfectly proper indicator.

# Annualized Net Present Value

## ANPV calculation:

| ANPV | Rate | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Total for the period |
|---|---|---|---|---|---|---|---|---|
| **NCF (Net Cash Flow) after tax** | | **-5 000** | **3 845** | **-300** | **6 920** | **8 691** | **10 477** | **24 633,16** |
| Compounding rate | ***12%*** | 1,0000 | 1,1200 | 1,2544 | 1,4049 | 1,5735 | 1,7623 | |
| Discount factor | ***12%*** | 1,0000 | 0,8929 | 0,7972 | 0,7118 | 0,6355 | 0,5674 | **3,60** |
| **NPV** | | | | | | | | **14 587,73** |
| **ANPV (Annualized Net Present Value)** | | | **4 047** | **4 047** | **4 047** | **4 047** | **4 047** | **4 046,78** |
| Calculated Annuity discounted | | | 3 613,19 | 3 226,07 | 2 880,42 | 2 571,80 | 2 296,25 | **14 587,73** |

**4 046,78** Calculated using "**ПЛТ**" function, English version "**PMT**"

### Syntax

**=-ПЛТ(Discount Rate;Number of period (since Y1);NPV;;0)**

# Unequal Lives Projects

Sometimes in order to compare several Projects with unequal duration the analysts use following ratios:

1) **Average NPV** (aNPV) = NPV / Number of periods. This ratio gives not exactly adequate valuation. For instance, the project with less aNPV gives larger inflows at the very beginning of the project. And these cash flows can be re-invested for longer period.
2) **Relative NPV** (rNPV) = NPV / (Number of periods * Initial investment). This ratio reflects how much **Value added** is generated **by each dollar of investment per period**. rNPV is a quite useful ratio which allows to compare the project with both: unequal maturity and unequal investment amounts. But normally, it's used in combination with other "classic" ratios.

| | Y0 | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 | NPV | ANPV/EAA | aNPV | rNPV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Project A** | -100 | 115 | | | | | | | | 4,13 ₽ | 4,55 ₽ | 2,07 ₽ | 2,066% |
| **Project B** | -150 | 30 | 30 | 20 | 20 | 20 | 10 | 30 | 90 | 5,63 ₽ | 1,06 ₽ | 0,63 ₽ | 0,417% |
| ***Discount rate*** | ***10%*** | | | | | | | | | | | | |

# Unequal Lives Projects

**ANPV** is mainly used for the comparison of the Projects with unequal maturities.

| Unequal lives projects comparison | | | | | |
|---|---|---|---|---|---|
| | | Y1 | Y2 | Y3 | IRR |
| **Project A** | -100 | 60 | 90 | 0 | **29,50%** |
| **Project B** | -140 | 80 | 70 | 60 | **24,76%** |
| Number of period | 1 | 2 | 3 | 4 | 5 |
| Discount Factor | 1,00 | 0,91 | 0,83 | 0,75 | ***10,00%*** |
| | | | | | **NPV** |
| **Project A** | -100 | 55 | 74 | 0 | **28,93** |
| **Project B** | -140 | 73 | 58 | 45 | **35,66** |

If we compare 2 Projects with 2- and 3-years maturities using traditional techniques – the Project B has **larger NPV** and should be graded higher.

# Unequal Lives Projects

We can use **Least Common Multiple of Lives** Approach. We extend the time horizon of analysis. It's supposed that each 2 or 3 years each project is re-implemented till the time when both projects are completed at the same date.

| Least Common Multiple of Lives | | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | |
|---|---|---|---|---|---|---|---|---|
| **Renovation of the Project each last year of the cycle** | 1,0000 | 0,9091 | 0,8264 | 0,7513 | 0,6830 | 0,6209 | 0,5645 | **NPV** |
| | | | *(-100+90)* | | *(-100+90)* | | | |
| **Project A** | -100 | 60 | -10 | 60 | -10 | 60 | 90 | **72,59** |
| **Project B** | -140 | 80 | 70 | -80 | 80 | 70 | 60 | **62,45** |
| | | | | *(-140+60)* | | | | |

**Least Common Multiple** method gives an opposite result – the Project A has **larger NPV** and should be graded higher.

# Unequal Lives Projects

We can also use ANPV or **Equivalent Annual Annuity Approach**.

| Equivalent Annual Annuity | EAA | | |
|---|---|---|---|
| Project A | 16,67 | | |
| Project B | 14,34 | Project B is worse | |

**ANPV** method gives an opposite result – the Project A has **larger ANPV** and should be graded higher.

The final comparison

| Projects comparison | Project A | Project B |
|---|---|---|
| Direct NPV approach | 28,93 | 35,66 |
| Least Common Multiple of Lives Approach | 72,59 | 62,45 |
| ANPV / EAA Approach | 16,67 | 14,34 |

# Investment Decision Criteria

The **NPV**, **IRR**, **PP**, **DPP**, **ANPV** and **DPI** are the most often used ratios for the initial selection of the Projects. All other measures should be also used but the key differentiation of the Projects is based on 6 main ratios. And, certainly, the **Net Present Value** is an essence of the **Investment Projects Analysis**.

## Project Selection

| | | | Discount rate | *10%* | | |
|---|---|---|---|---|---|---|
| **Year** | **Project A** | **Project B** | **Project C** | **Project D** | **Project E** | **Project F** |
| 0 | -1 000 | -1 000 | -1 000 | -1 000 | -1 000 | -1 000 |
| 1 | 1 000 | 100 | 400 | 500 | 400 | 500 |
| 2 | | 200 | 300 | 500 | 400 | 500 |
| 3 | | 300 | 200 | 500 | 400 | 10 000 |
| 4 | | 400 | 100 | 500 | 400 | |
| 5 | | 500 | 500 | | 400 | |
| 6 | | | | | | |
| 7 | | | | | | |
| **PBP** | 1,0 | 4,0 | 4,0 | 2,0 | 2,5 | 2,0 |
| **NPV** | -90,91 | 65,26 | 140,60 | 584,93 | 516,31 | 7 380,92 |
| **IRR** | 0,00% | 12,01% | 15,52% | 34,90% | 28,65% | 141,77% |

| **Comments** | | | |
|---|---|---|---|
| NPV negative. Even no accounting profit. | The same PBP but NPV & IRR of Proj C are larger because of earlier higher repayments | The shorter PBP, anyway, has smaller NPV & IRR | PBP dissembles after payback date incomes. |