По материалам сайта: [www.pascalroussel.net/?p=834](http://www.pascalroussel.net/?p=834)

[Valuation of swaps](http://www.pascalroussel.net/?p=834)

VALUATION OF IRS

An IRS could be valued in 2 different ways giving the same result:

Valuation in terms of bond prices

As shown in the section LIBOR/swap zero rate, a swap could be seen as an exchange of bonds:



Let’s note F the principal notional

Let’s consider the dealer’s perspective:

* He has a short position in a fixed rate bond corresponding to payments that are made (the dealer is selling fixed rate and as a payment he is receiving floating rate). Let’s note Bfixthe value of that fixed rate bond.
* He has a long position in a floating rate bond corresponding to payments that are received. Let’s note Bfloat the value of that floating rate bond.
* The dealer is getting from the customer a bond having a value Bfloat and giving him a bond with value Bfix .The value of the swap is Vswap\_dealer = Bfloat – Bfix

The customer’s perspective is exactly the opposite and for him the value of the swap is Vswap\_customer = – Vswap\_dealer =  Bfix  – Bfloat

Calculation of the current value of Bfix

Let’s note:

* rfix the agreed fixed rate p.a. which could be compounded “m” times per annum.
* ri the Libor/swap zero rate p.a. with continuous compounding covering the period from today to time “ ti “ when a swap payment is performed.
* “ n “ the last payment

After discounting all the future cash flows, we have:



Calculation of the current value of Bfloat

Let’s consider the initial time, t = 0, the time corresponding to one payment of both swap legs.

Let’s note F, the face value of the floating rate bond. As seen, the value of a floating rate bond immediately after the payment of its coupon, is its face value “F”.

We have

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| t = 0 |  | just before t\* | t = t\* | Immediately after t\* |
| * Libor is fixed for the next payment to be made at t = t\*
* Based upon that libor rate, the floating payment will be “ fl\* “
 |  | Bfloat  = F + fl\* | The floating payment “ fl\* “ is done | Bfloat  = F |

The floating-rate bond can therefore be regarded as an instrument providing a single cash flow of F + fl\* at time t\*.
The current value of that bond is obtained by discounting that single cash flow:



with

r1= the Libor/swap zero rate p.a. with continuous compounding covering the period from today to time “ t\* “ when the next swap payment is performed.

fl\* = the next amount that will be paid at time t\*, by the floating rate payer

 Valuation in terms of FRAs

As already seen in the section “FRA seen as a swap”, a swap could be considered as a portfolio of forward rate agreements.

As previously seen in the section “valuation of FRA”, we can value a FRA by assuming the forward interest rates are realized so that the future value of the LIBOR rate = the forward rates.

As seen in the section “valuation of FRA”, the forward rates could be determined once the[LIBOR/swap zero curve](http://www.pascalroussel.net/swap.htm#swap_138) is established

The valuation in terms of FRA is a 3 steps process:

1. The first step:
is to determine the forward rates by using the LIBOR/swap zero curve and calculate the future floating rate payments by assuming that LIBOR rate = the forward rates. The forward **continuous** rate must be converted into a **compound** interest rate to be used in the calculation of the floating payments in step 2
2. The second step:
is to calculate the future swap cash flows (for the dealer above it is “floating payments – fixed payment”).
3. The third step:
is to discount those cash flows by using the LIBOR/swap zero curve.

**Important note**: an important question for the dealer is how to calculate the value of the fixed rate ? the answer is simply by calculating it so that the swap is worth zero initially. Based upon that value, he can add his fee.

Example of IRS valuation

Let’s consider the following swap (semiannual compounding):

Notional = 150 MUSD

The Libor rate is paid by the customer

The Fixed rate is paid by the dealer and is: 7% p.a. or 3.5% per 6-month



Payment schedule:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3 months ago< | **today** | 3-month = 0.25 yr = t\* | 9-month = 0.75 yr | 15-month = 1.25 yr |
| Libor rate agreed at 9.3% p.a. |  | next payment | payment | last payment |

The Libor/swap zero continuous rates p.a. are :

|  |  |  |
| --- | --- | --- |
| 3-month = 0.25 yr | 9-month = 0.75 yr | 15-month = 1.25 yr |
| r1 = 9% | r2 =9.5% | r3 =10.8% |

Valuation in term of bond prices

Calculation of the current value of Bfix :



 MUSD

Calculation of the current value of Bfloat :

 MUSD

Therefore



 MUSD

For the dealer the value of the swap is

Vswap\_dealer = Bfloat- Bfix  = 153.483-145.667 = **7.816 MUSD**

Valuation in term of FRAs

First step:

Since we must assume that the unknown future floating rate will be the forward rate, we must first calculate them:

Regarding the next payment in 0.25 year, the semiannually compounded rate has previously been fixed at 9.3% p.a.

Regarding the payment in 0.75 year we need to calculate the forward rate for the period between 0.25 year and 0.75 year.

We have the **continuous forward rate RC\_F**:



Since the floating payment is a semiannual compounding, we need to convert the continuous rate into a semiannual compounding rate by using the [conversion formula](http://www.pascalroussel.net/financial_mathematics.htm) :



In the same way, we can determine that the semiannual compounding forward rate for the period from 0.75 year to 1.25 year is 13.17%

Second step:

Calculation of the future swap cash flows (in MUSD), for the dealer:

|  |  |  |  |
| --- | --- | --- | --- |
| Time | Fixed cash flow | Floating cash flow | Δ=Floating – Fixed |
| 0.25 | 150 \*3.5% = 5.25 | 150\*9.3%/2 = 6.975 | 1.725 |
| 0.75 | 150 \*3.5% = 5.25 | 150\*9.99%/2 = 7.494 | 2.244 |
| 1.25 | 150 \*3.5% = 5.25 | 150\*13.17%/2 = 9.873 | 4.624 |

Third step:

Discounting those cash flows (in MUSD):

|  |  |  |  |
| --- | --- | --- | --- |
| Time | Δ=Floating – Fixed | Discounting factor | Present Value |
| 0.25 | 1.725 | http://www.pascalroussel.net/images/valuat73.gif | 1.687 |
| 0.75 | 2.244 | http://www.pascalroussel.net/images/valuat74.gif | 2.089 |
| 1.25 | 4.624 | http://www.pascalroussel.net/images/valuat75.gif | 4.040 |
|  |  | TOTAL | **7.816** |

VALUATION OF CURRENCY SWAPS

In the same way as described in the section related to the valuation of an IRS here above, a currency swap could be valued in 2 different ways giving the same result:

Valuation in terms of bond prices

A currency swap could be seen as an exchange of bonds:



Let’s consider the dealer’s perspective:

* He has a short position in a bond corresponding to payments that are made in currency A. Let’s note Bcurrency A the value (in currency A) of that bond.
* He has a long position in a bond corresponding to payments that are received in currency B. Let’s note Bcurrency B the value (in currency B) of that bond.
* The value of the swap is therefore: Vswap\_dealer = Bcurrency B  -  Bcurrency A

However we cannot subtract one currency from an other !
One of the bond value has to be converted in the currency of the other bond. For example, the value in currency A of the swap is:

Vswap\_dealer\_in currency A =SA/1B \* Bcurrency B- Bcurrency A

The spot exchange rate is given as “currency A per currency B”:

SA/1B the spot exchange rate such that “SA/1B“ amount of currency A  = 1 currency B (e.g. if currency A is USD and currency B is €, we could have SA/1B = 1.2 such that  1.2 USD = 1 €).

The customer’s perspective is exactly the opposite and for him the value of the swap is – Vswap\_dealer

Calculation of the current value of the bond

Let’s note for the leg in currency A:

* rfix\_A the agreed fixed rate p.a. in currency A which could be compounded “m” times per annum.
* rA\_i the Libor/swap zero rate p.a. in currency A with continuous compounding covering the period from today to time “ ti “ when a swap payment is performed .
* “ n “ the last payment
* FA the principal notional in currency A

After discounting all the future cash flows, we have the bond value:



Let’s note for the leg in currency B:

* rfix\_B the agreed fixed rate p.a. in currency B which could be compounded “m” times per annum.
* rB\_i the Libor/swap zero rate p.a. in currency B with continuous compounding covering the period from today to time “ ti “ when a swap payment is performed .
* “ n “ the last payment
* FB the principal notional in currency B

After discounting all the future cash flows, we have the bond value:



Valuation in terms of forward contracts

We have seen in a previous section “Futures and Forward contracts on currencies” that the forward exchange rates could be determined once the LIBOR/swap zero curve is established.

We have the relationship between the forward exchange rate applicable in N years and the current spot exchange rate:



With

* rA is the continuous risk free rate of currency A applicable from today to N years
* rB is the continuous risk free rate of currency B applicable from today to N years
* the exchange rate is given as “currency A per currency B”: SA/1B the spot exchange rate such that “SA/1B“ amount of currency A  = 1 currency B

We have seen also that we can value a forward foreign exchange contract by assuming that the forward exchange rates are realized so that the future value of the spot exchange rates = the forward exchange rates.

And finally we have already seen in the section “FRA seen as a swap”, that a swap could be considered as a portfolio of forward contracts.
The valuation in terms of forward contracts is therefore a 3 steps process:

1. The first step:
is to determine the forward exchange rates by using the current spot exchange rate and the LIBOR/swap zero curve and calculate the future exchange rate payments by assuming that the future value of the spot exchange rates = the forward exchange rates.
2. The second step:
is to calculate the future swap cash flows (for the dealer above it is “currency B payments – currency A payments”).
3. The third step:
is to discount those cash flows by using the LIBOR/swap zero curve.

**Important note**: an important question for the dealer is how to calculate the value of the fixed rate in currency A? The answer is simply by calculating it so that the swap is worth zero initially. Based upon that value, he can add his fee.

If the initial value of a swap is zero, it could be demonstrated that for the payer of the lowest rate the swap value is negative during almost the entire swap life. This could be important in regard to credit risk discussed in the section “swap risk“.

Example of currency swap valuation

Let’s consider the following swap (semiannual compounding):



Payment schedule:

|  |  |  |  |
| --- | --- | --- | --- |
| **today** | 3-month = 0.25 yr | 9-month = 0.75 yr | 15-month = 1.25 yr |
|  | Next payment | Payment | Last payment |

Currency A is USD:

Notional in USD : FA = 15 MUSD

The fixed rate in USD paid by the dealer is: 7% p.a. or 3.5% per 6-month

The Libor/swap zero continuous rates p.a. for USD are:

|  |  |  |
| --- | --- | --- |
| 3-month = 0.25 yr | 9-month = 0.75 yr | 15-month = 1.25 yr |
| rA\_1 = 9% | rA\_2 =9.5% | rA\_3 =10.8% |

Currency B is YEN:

Notional in YEN : FB = 1500 MYEN

The fixed rate in YEN received by the dealer is: 5% p.a. or 2.5% per 6-month

The swap zero continuous rates p.a. for YEN are:

|  |  |  |
| --- | --- | --- |
| 3-month = 0.25 yr | 9-month = 0.75 yr | 15-month = 1.25 yr |
| rB\_1 = 4% | rB\_2 =4.5% | rB\_3 =4.8% |

Current spot exchange rate:

SA/1B = 0.00909    (0.00909 USD = 1 YEN)

Valuation in term of bond prices

Calculation of the current value of Bcurrency\_A :



 MUSD

Calculation of the current value of Bcurrency\_B :



 MYEN

For the dealer the value of the swap in USD is

Vswap\_dealer\_in currency A = SA/1B \* Bcurrency B- Bcurrency A

Vswap\_dealer\_in currency A = 0.00909\*1521.345 – 14.567 = **-0.736 MUSD**

Valuation as a portfolio of forward contracts

First step:

Since we must assume that the unknown future exchange rates will be the forward exchange rates, we must first calculate them:

Regarding the next payment in 0.25 year:



Regarding the payment in 0.75 year:



Regarding the last payment in 1.25 year:



Second step:

Calculation of the future swap cash flows for the dealer:

|  |  |
| --- | --- |
| Time | Cash flow in currency A (in MUSD) |
| 0.25 | 15\*3.5% = 0.525 |
| 0.75 | 15\*3.5% = 0.525 |
| 1.25 | 15+15\*3.5% =15.525 |

|  |  |  |
| --- | --- | --- |
| Time | Cash flow in currency B (in MYEN) | Cash flow in currency B converted in MUSD |
| 0.25 | 1500\*2.5% = 37.5 | 37.5 \* 0.0092 = 0.3452 |
| 0.75 | 1500\*2.5% = 37.5 | 37.5 \* 0.0094 = 0.3539 |
| 1.25 | 1500+1500\*2.5% = 1537.5 | 1537.5 \* 0.0098 = 15.0659 |

|  |  |
| --- | --- |
| Time | Δ = cash flow in currency B converted  – cash flow in currency A |
| 0.25 | 0.3452 – 0.525 = -0.18 |
| 0.75 | 0.3539 – 0.525 = -0.171 |
| 1.25 | 15.0659 – 15.525 = -0.459 |

Third step:

Discounting those cash flows (in MUSD):

|  |  |  |  |
| --- | --- | --- | --- |
| Time | Δ | Discounting factor | Present Value |
| 0.25 | -0.18 | http://www.pascalroussel.net/images/valuat73.gif | -0,1758 |
| 0.75 | -0.171 | http://www.pascalroussel.net/images/valuat74.gif | -0,1593 |
| 1.25 | -0.459 | http://www.pascalroussel.net/images/valuat75.gif | -0,40114 |
|  |  | TOTAL | **-0,73624** |

VALUATION OF AN ASSET SWAP

There is basically no major difference between the valuation of an [asset swap](http://www.pascalroussel.net/swap.htm#asset_swap) and an IRS.

Example of an asset swap valuation in terms of FRA

In this example we will show how the spread is calculated. The basic idea is to say that at the beginning the asset swap value should be null.



Let’s consider the following bond owned by company B and that will be swapped “[not discounted](http://www.pascalroussel.net/swap.htm#asset_swap)” :

Face value: 100 EUR

Coupon rate: 5%

Compound: semi annually

Price: 90 EUR

Life: 5 years

Let’s imagine that the EURIBOR/Swap rates are :

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Continuous rate p.a. | 5,95% | 5,96% | 5,98% | 6,02% | 6,09% | 6,18% | 6,30% | 6,46% | 6,66% | 6,91% |

Present value of the coupon leg:

Since the bond price is below par, company B must pay at the beginning of the swap (“not discounted “ type):

100 – 90 = 10 EUR

The coupon payment is:

100\* 5%/2 =2.5 EUR

We can then calculate the present value of the fixed rate leg:

>2,15>2,01

|  |  |  |  |
| --- | --- | --- | --- |
| Time in year | Payment | Discount factor | Present value |
| 0 | 10 | 1 | 10 |
| 0.5 | 2.5 | http://www.pascalroussel.net/images/valuat89.gif | 2,43 |
| 1.0 | 2.5 | http://www.pascalroussel.net/images/valuat90.gif | 2,36 |
| 1.5 | 2.5 | http://www.pascalroussel.net/images/valuat91.gif | 2,29 |
| 2.0 | 2.5 | http://www.pascalroussel.net/images/valuat92.gif | 2,22 |
| 2.5 | 2.5 | http://www.pascalroussel.net/images/valuat93.gif | 2,15 |
| 3.0 | 2.5 | http://www.pascalroussel.net/images/valuat94.gif | 2,08 |
| 3.5 | 2.5 | http://www.pascalroussel.net/images/valuat95.gif | 2.01 |
| 4.0 | 2.5 | http://www.pascalroussel.net/images/valuat96.gif | 1,93 |
| 4.5 | 2.5 | http://www.pascalroussel.net/images/valuat97.gif | 1,85 |
| 5.0 | 100+2.5 | http://www.pascalroussel.net/images/valuat98.gif | 72,56 |
|  |  | TOTAL | 101,85 EUR |

Present value of the variable leg:

We need first to estimate the future 6-month Euribor rates that we will be used each time a swap payment is performed.

As seen in the section “valuation of a FRA”, the forward rates could be determined from the above given  LIBOR/swap zero curve.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  year | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| period in month | 0-6 | 6-12 | 12-18 | 18-24 | 24-30 | 30-36 | 36-42 | 42-48 | 48-54 | 54-60 |
| continuous forward rate | 5,95% | 5,97% | 6,03% | 6,14% | 6,335% | 6,625% | 7,03% | 7,58% | 8,28% | 9,16% |
| equivalent rate compounded semiannuallay | 6,04% | 6,06% | 6,12% | 6,24% | 6,44% | 6,74% | 7,16% | 7,72% | 8,45% | 9,37% |

Let’s note S% the unknown spread p.a.

|  |  |  |  |
| --- | --- | --- | --- |
| Time in year | Payment | Discount factor | Present Value |
| 0.5 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 1.0 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 1.5 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 2.0 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 2.5 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 3.0 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 3.5 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 4.0 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 4.5 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
| 5.0 | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif | http://www.pascalroussel.net/images/valuat103.gif |
|  |  | TOTAL | Should be 101.85 EUR |

We must choose a value for S% such that the present value of both legs are equal. In that case S=0.44% per 6-month or 0.88% p.a. semi annual compounding.